

# Two-Way Capital Flows and Global Imbalances \*

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This paper shows how underdeveloped financial markets in emerging economies can explain the pattern of two-way capital flows between emerging economies (such as China) and the developed world (such as the United States). Our calibrated model reproduces China's rising financial capital outflows and FDI inflows as well as its massive trade imbalances in the past decades. Our model also predicts that (i) global trade imbalances may be sustainable even in the long run and (ii) the conventional wisdom that the "saving glut" of emerging economies is responsible for the global low interest rate may be wrong.

The pattern of international capital flows is a long-standing puzzle. Lucas (1990) ponder why capital does not flow from North (developed countries) to South (developing countries) even though it is scarcer and commands a higher rate of return (or marginal product) in the latter. The standard neoclassical growth theory attributes the high marginal product of capital (MPK) in the South to low household savings, thus predicting a capital flow from rich to poor countries. But in fact savings abound in many emerging economies and massive amount of capital have been flowing into rich countries over the past decades.

To explain the "reverse" capital flow puzzle, the mainstream literature on global imbalances argues that the rate of return to capital is actually lower (rather than higher) in developing economies because of a savings glut (Bernanke, 2005). Hence, capital moves in the reverse direction—from South to North.

However, the reverse capital flow puzzle is partially a fallacy of aggregation. In reality, *fixed* capital does flow mainly from North to South, in the form of foreign direct investment (FDI). It is *financial* capital (portfolio investment) that has been flowing in the opposite direction. Since historically the "uphill" flows of financial capital dominate the "downhill" flows of fixed capital, the net aggregate capital flow (financial plus fixed) shows the reverse pattern.

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For example, during the 2000-2011 period, industrial countries as a whole had net financial capital inflows (including foreign reserve decumulations) averaging -\$498 billion per year and net FDI outflows averaging \$295 billion per year. In contrast, the less developed countries (LDCs) as a block had net FDI inflows averaging -\$246 billion per year and net financial capital outflows (including foreign reserve accumulations) averaging \$354 billion per year. These opposite movements (or diverging trends) in financial and fixed capital flows have been growing over time. In the meantime, industrial countries have been running large and persistent trade deficits with the South. The major countries contributing to such global imbalances are the U.S. (representing developed countries) and China (representing LDCs in recent years). In particular, China is now both the largest holder of foreign reserves (more than \$3 trillion by the end of 2011, mostly U.S. government bonds) and the largest recipient of FDI (more than \$1.4 trillion by the end of 2011) among developing countries, as well as the main contributor to global current account imbalances (e.g., with average surplus of over \$250 billion per year in the 2005-2011 period). In contrast, the U.S. is the largest importer of financial capital from developing countries and the largest exporter of FDI to the South. Meanwhile, the U.S. is also the country with the largest trade deficit (e.g., with average current account deficit of over \$600 billion per year in the 2005-2011 period).

Despite the importance of FDI in North-South trade and its growing significance in rebalancing international capital flows and national current accounts, the bulk of the existing literature on global imbalances does not distinguish financial capital from fixed capital flows. Failing to distinguish these two forms of capital flows not only obscures the reality but may also impede correct theoretical analysis and empirical testing with different models aimed at explaining capital flows and the associated global imbalances.

This paper provides a framework to explain the two-way capital flow puzzle by augmenting the neoclassical growth model with financial frictions under incomplete markets. Specifically, following the approach of Gourinchas and Jeanne (2013), we augment the neoclassical growth model with two wedges: one that distorts firms' investment decisions and another that distorts households' saving decisions. However, unlike in Gourinchas and Jeanne's (2013) approach where the wedges are ad hoc black boxes, in our approach these wedges are explicitly derived through financial frictions, thus providing micro foundations for these theoretical constructs.

Our story proceeds as follows. Due to an underdeveloped banking-credit-financial system, both households and firms in the South are severely borrowing constrained. As a result, households opt to save excessively to self-insure against unpredictable shocks, and firms have to rely heavily on internal cash flows to finance fixed investment. Since domestic savings by households cannot be effectively channeled to firms where fixed capital formation takes place, fixed capital is scarce in the production sector while savings are abundant in the household sector. In such a world, the rate of

return to financial assets can be significantly lower than that of fixed capital. In China, for example, the real rate of return to fixed capital has consistently been over 20% in the past decades while the real rate of return to financial capital (such as bank deposits and short-term bonds) has been negative (Bai, *et al.*, 2006). Despite such an enormous gap, households in China save excessively and the bulk of their savings is in the form of bank deposits (Wen, 2009). This enormous arbitrage opportunity implies that financial liberalisation between the South and the North will trigger two-way capital flows. Because it is relatively easier for financial capital to flow internationally than for fixed capital to be shipped abroad (e.g., due to transaction and transportation costs), the former will dominate the latter in global capital flows, resulting in short-run current account imbalances. In addition, because the rates of return to fixed and financial capital differ, the net income (interest) payments on the opposite capital flows do not cancel out, further contributing to global trade imbalances even in the long run.

Therefore, in contrast to the standard neoclassical theory which attributes high MPK in the South to low household savings, we show how the lack of an efficient financial system in the South can lead to insufficient investment on the firm side but a saving glut on the household side, resulting in a high MPK and a low interest rate at the same time. These wedges in rates of return drive the observed two-way capital flows between developing and developed countries and the current account imbalances. More importantly, we show that such two-way capital flows can sustain permanent trade imbalances even if the current account is perfectly balanced at zero.<sup>1</sup>

Our analysis is related to a large and growing literature on global imbalances. Ju and Wei (2010) study two-way capital flows in a static non-neoclassical model with a focus on corporate governance and property rights. Caballero *et al.* (2008) attribute the global imbalances to the South's inability to generate saving instruments, leading to the reverse capital flow after financial liberalisation. Mendoza, *et al.* (MQR, 2009) blame the global imbalances on the heterogeneous degrees of financial development between developed and developing countries. Such heterogeneity implies that households in the North prefer riskier equity in their portfolios than do households in the South, causing the South to maintain a positive net asset position in risk-free bonds. Similar to MQR (2009), Angeletos and Panousi (2011) ascribe global imbalances to heterogeneous degrees of idiosyncratic risks between the North and the South. Outflows of financial capital from the South are driven by its low interest rate under the precautionary saving motives. Like us, Angeletos and Panousi (2011) allow firms to accumulate fixed capital and their model can also generate a wedge between the MPK and the real interest rate. In contrast to our study, however, theirs does not consider FDI and two-way capital flows. Related works also include Ohanian and Wright (2007),

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<sup>1</sup>That is, imbalanced trade exists even if financial capital flows and fixed capital flows exactly cancel (balance) each other—because the cross-country net factor payments do not necessarily cancel each other due to the investment wedge.

Sandri (2010), Carroll and Jeanne (2009), Durdu, *et al.* (2009), Buera and Shin (2010), Chien and Naknoi (2011), Song, *et al.* (2011), Gourinchas and Jeanne (2013), Andolfatto (2012), and Wen (2009, 2011), among many others.<sup>2</sup>

However, the bulk of this literature does not distinguish between financial capital and fixed capital flows.<sup>3</sup> As such, many of the models that are proposed to explain the global imbalances are simply inconsistent with the empirical pattern of the two-way capital flows and trade imbalances. Typically, because no distinction is made between household savings and firms' fixed capital stocks, to explain the reverse capital flow such a model would imply excess domestic savings in the form of tangible capital goods, which are rented out to foreign firms as a form of capital outflows (e.g., Carroll and Jeanne, 2009). This particular form of capital outflows from the South to the North is inconsistent with the empirical facts.<sup>4</sup>

The work closest to ours is MQR (2009). Our approach complements that of MQR in several aspects. In contrast to our full-fledged dynamic model, MQR's model assumes that the stock of aggregate capital is fixed in each country and there is no labor market, and so there are no cross-country fixed capital flows by assumption.<sup>5</sup> Most importantly, FDI is modeled by MQR as purchases of foreign firms' equities. While foreign equity holding is a special form of FDI, it is no longer the major form of FDI. Data show that the currently dominant form of FDI involves setting up new firms or establishing new affiliations in foreign countries by exporting technology-embodied fixed capital and receiving factor payments as capital owners. For example, based on the total nonfinancial capital outflows from the U.S. to the rest of the world (ROW), the particular form of FDI assumed in the model of MQR (2009) accounts for less than 38% of total FDI, leaving more than 62% of U.S. FDI unexplained. In contrast, the specific form of FDI studied in our paper accounts for more than 76% of U.S. FDI outflows to China. Also, the new establishment of affiliations (or firms) with ownership fully belonging to foreigners accounts for 80% of China's total inward FDI from developed countries in 2009 and 2010 and this number is still growing.<sup>6</sup> Therefore, our approach represents a big step toward understanding the mechanisms of FDI and its role in global imbalances.

Moreover, the model of MQR generates a trade surplus for the U.S. in the longer term. In their model the interest payment on the inflow of financial capital from developing countries outweigh

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<sup>2</sup>This literature does not address the main issues raised by Wen (2011), especially the positive relationship between China's high saving rate and rapid income growth rate and the connection between capital controls and trade.

<sup>3</sup>Even in the model of Ju and Wei (2010), there is only one form of capital that flows in and out to form a two-way flow circle. For example, it flows out to by-pass domestic regulations and then flows back. Capital flows in the form of FDI are not explicitly modeled.

<sup>4</sup>On the other hand, a model that can generate low interest rate through precautionary savings would also imply low MPK (i.e., Aiyagari, 1994), but in the data countries with saving gluts have high MPKs.

<sup>5</sup>However, they allow nonreproducible managerial capital or human capital to be reallocated across borders.

<sup>6</sup>See the online Data Appendix C.1 for details of the classifications and compositions of FDI in the U.S. and China.

the returns from outward FDI, and so the U.S. net foreign income payment is positive in the steady state. Hence, their model does not support the notion that the persistent U.S. trade deficits with China and the ROW may in fact be sustainable in the long run.<sup>7</sup>

The rest of the paper is organised as follows. Section 1 presents stylised facts about the two-way capital flows between China (representing the South) and the U.S. (representing the North). Section 2 introduces an extremely simple toy model to illustrate how our story can explain the stylised facts. Section 3 presents our full-fledged dynamic model. Section 4 studies the conditions for generating two-way capital flows. Section 5 provides quantitative predictions and Section 6 concludes the paper.

## 1 Stylised Facts

We decompose global capital flows into financial capital flows and nonfinancial capital (FDI) flows. We first use data from China to represent developing countries (South) and those from the U.S. to represent the developed world (North).<sup>8</sup> We begin with the following three observations.

- 1 *China (the U.S.) is a net exporter (importer) of financial capital and a net importer (exporter) of FDI.*<sup>9</sup>

Figure 1A shows the net foreign asset positions of the U.S. with respect to China. In particular, the dark line with a positive trend shows the accumulated net FDI outflows from the U.S. to China as a share of U.S. GDP (left axis), and the grey line with a negative trend shows the accumulated net financial capital inflows from China to the U.S. as a share of U.S. GDP (right axis). Figure 1B plots the net foreign asset positions of China against ROW (mostly developed countries). The grey line with an upward trend indicates China's total accumulated net financial capital outflows, which accounts for about 50% of the country's GDP in 2010. The dark line with a downward trend shows China's total accumulated net FDI inflows, which account for about 20% of the country's

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<sup>7</sup>In addition, the model of MQR rules out any aggregate risks to reduce computational burdens. Without aggregate uncertainty, their model generates only a small risk premium for the rate of return to FDI (i.e., holdings of foreign capital stocks), and this small risk premium leads to a positive net factor payment (interest payment minus FDI earnings). To overcome the computational challenge under aggregate risk, Chien and Naknoi (2011) simplify the MQR model to a pure endowment economy and use a special algorithm to numerically solve the model. They show that with aggregate uncertainty (stochastic output growth), the model can generate a large risk premium between equity and risk-free bonds and thus is able to generate long-term trade deficits for the U.S. However, their model is not suited for studying the two-way capital flows discussed in this paper because it is an endowment economy without capital. Our approach can easily handle any number of aggregate shocks without having to rely on aggregate risk to generate high returns to FDI because we consider an entirely different channel of FDI.

<sup>8</sup>Online Data Appendix C.1 provides details about the data series used in this section.

<sup>9</sup>Following Ju and Wei (2010), we define net FDI outflows = [FDI asset - FDI liability], and net financial capital outflows = [total foreign asset - FDI asset] - [total foreign liability - FDI liability]. This definition is equivalent to define net financial capital outflow = [portfolio equity assets - portfolio equity liabilities] + [debt assets - debt liabilities] + [financial derivatives assets - financial derivatives liabilities] + [foreign exchange reserves - gold].

GDP in 2010.<sup>10</sup>

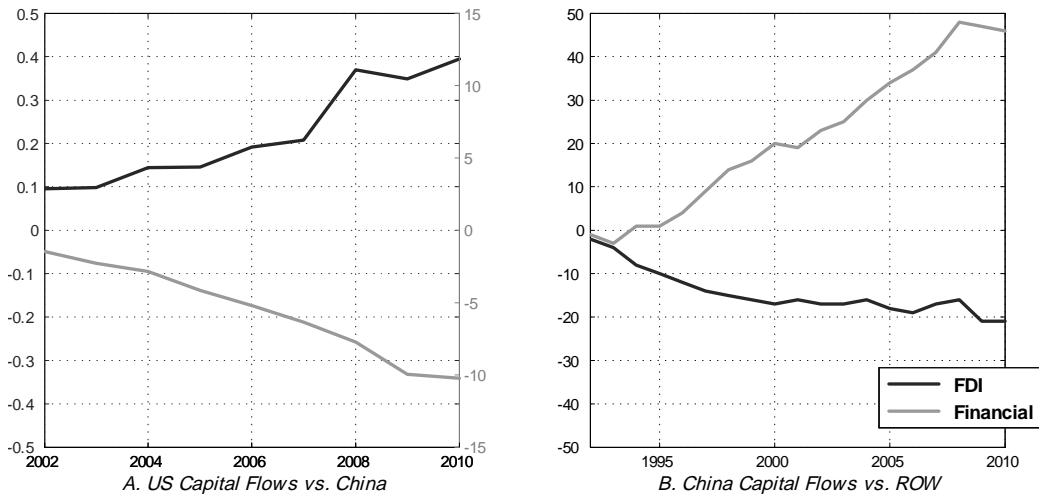


Figure 1. *Two-Way Capital Flows*

2 *China has a significantly higher rate of return to fixed capital and a significantly lower rate of return to financial capital than the U.S.*

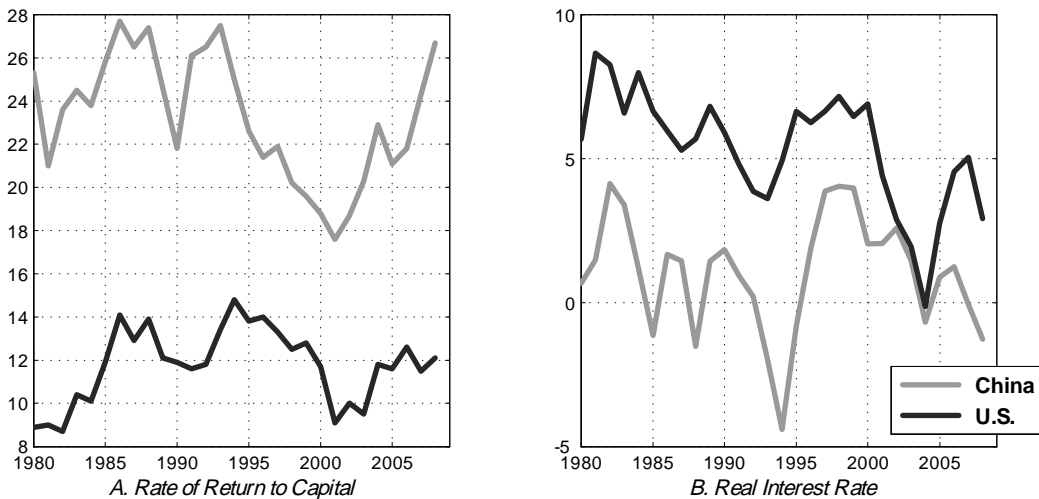


Figure 2. *Differential Asset Returns in China and U.S.*

Figure 2A compares the before-tax real rates of return to fixed capital in China (grey) and the U.S. (dark). China's capital return stayed at a very high level over the entire sample period, with a mean of 23% per year. In contrast, the rate of return to fixed capital in the U.S. was significantly

<sup>10</sup>Because China has been growing much faster than the ROW, its FDI inflows appear to have slowed in recent years relative to its GDP (Figure 1B). However, absolute magnitude has been accelerating. For example, the U.S. FDI to China does not show such a declining pattern as a share of U.S. GDP.

below that of China, with a mean of about 10% per year. The spread remained highly persistent over the entire sample period with only a slight decline in the mid-1990s.<sup>11</sup>

Table 1. *Real Interest Rates (Annual, 1990-2011)*

Period	1 Month	3 Month	6 Month	1 Year	2 Year	3 Year	5 Year
	China (Inflation Rate = 4.78%)						
Deposit rate (%)	-3.60	-1.79	-0.93	-0.13	0.44	1.01	1.62
Govt. bond (%)		-2.67	-2.58	-1.88		-1.77	-1.35
	U.S. (Inflation Rate = 2.75%)						
CD (%)	1.07	1.15	1.26				
T-bill (%)		0.69	0.80	1.18			2.06

Figure 2B shows that there is also a systematic difference in the rates of return to financial capital between the two countries, but the spread is reversed. For example, the annual real interest rate (defined as the risk-adjusted annual lending rate) in the U.S. (dark) is about 6% (per unit of risk) on average, whereas that in China (grey) is about 1% (per unit of risk) on average. Table 1 also shows a systematic cross-country gap of about 3 percentage points in the real interest rates (not adjusted for risk) when bank deposit rates and government bond rates are compared.<sup>12</sup>

### 3 *China has a less developed financial market than the U.S.*

Figure 3 shows that private credit-to-GDP ratios in both China and the U.S. have been rising gradually over time, which may indicate financial improvement in both countries. However, the disparity between the two countries is large and shows no signs of diminishing over time. We obtain similar results when using other measures of financial development.<sup>13</sup>

The two-way capital flow pattern shown in Figure 1 also exists in other major emerging economies and developed countries.<sup>14</sup> We first look at the pattern for the developed world, using data from 21 developed countries analysed by Ju and Wei (2010).<sup>15</sup> Figure 4A shows that the

<sup>11</sup>The after-tax rate of return in China was about 18% whereas that in the U.S. was about 7%. Therefore, even after taking tax adjustments into account, the rates of return to fixed capital in the two countries were still significantly different. We also calculated the U.S. rate of return to fixed capital through Poterba's (1998) method but the result changes little.

<sup>12</sup>Risk adjustment means dividing the interest rate by the relative standard deviations. The real rates in Table 1 are computed using the CPI inflation rate in each country. U.S. data are from FRED (Federal Reserve Bank of St. Louis). Chinese data are from the People's Bank of China.

<sup>13</sup>We follow the existing literature (e.g., King and Levine, 1993) by using the total private credit-to-GDP ratio as a measure of financial development because this variable captures the ability of financial intermediaries to allocate credit. A persistently higher ratio thus indicates a better financial system. The online Data Appendix C.2 presents more detailed empirical evidences on the measured gap of financial development between China and the U.S.

<sup>14</sup>The definitions of capital flows are the same as those in footnote 9. The series are measured in billions of dollars. The data set was updated from Lane and Milesi-Ferretti (2007) and the sample period is from 2000 to 2011.

<sup>15</sup>These countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, U.K., and the U.S.

developed countries as a whole, just like the U.S., exhibits a very significant two-way flow pattern, with net financial capital inflows (grey line) and net FDI outflows (dark line). Also, the net total asset position for the developed world is negative (dashed line), suggesting the reverse capital-flow pattern. We next look at the pattern for the emerging markets, using data from 22 emerging economies analysed by Ju and Wei (2010).<sup>16</sup> Figure 4B shows an upward trend in net financial capital outflows (grey line) and a negative trend in net FDI inflows (dark line), just the opposite of the trends in developed world.

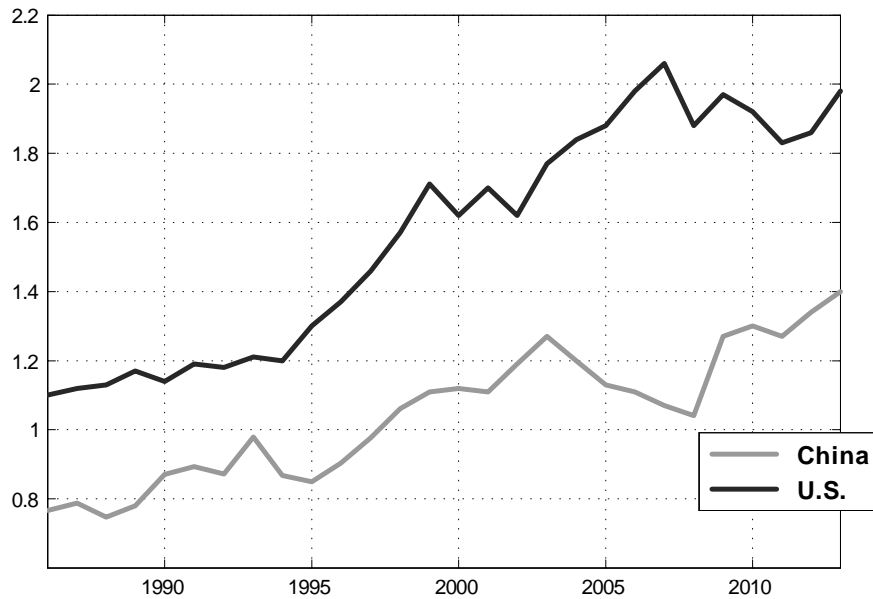


Figure 3. *Financial Development in China and U.S.*

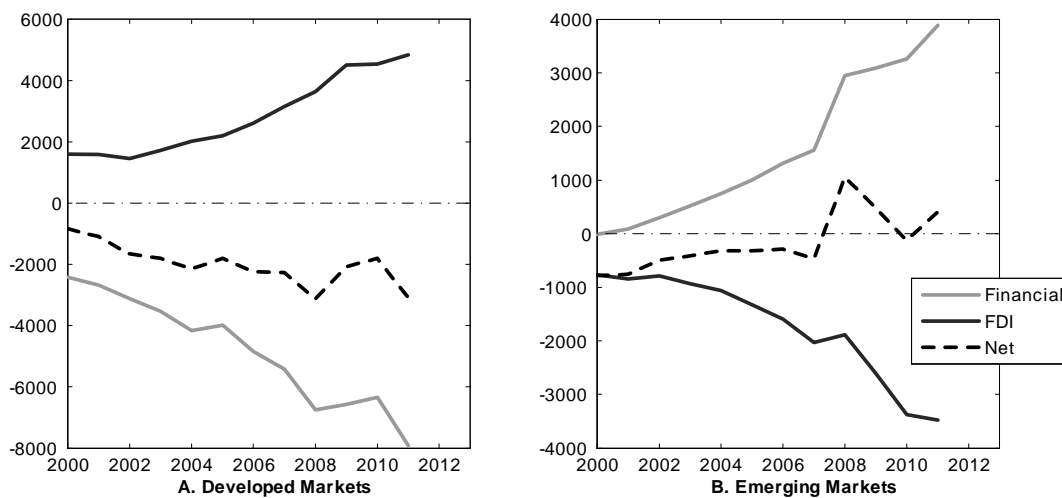


Figure 4. *International Two-Way Capital Flows*

<sup>16</sup>This country group includes Argentina, Brazil, Chile, China, Colombia, Egypt, Hong Kong SAR, India, Indonesia, Israel, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Singapore, South Africa, Thailand, Turkey, and Venezuela.



## 2 A Toy Model

We present first an extremely simple, two-country general-equilibrium toy model to illustrate the intuition and main thrust of our story, before proceeding to a more sophisticated full-fledged dynamic general equilibrium model in the next section (for calibration and quantitative exercises). The toy model economy has two countries, labeled  $h$  (home) and  $f$  (foreign). Each country is populated by a continuum of heterogeneous households indexed by  $i \in [0, 1]$ , and a representative firm. The foreign country is frictionless and the home country has financial frictions. We focus on the home country and drop the country index unless confusion may arise.

**Households.** Households live for two periods, period 1 and 2. They each supply inelastically one unit of labor in period 2 and consume in both periods, the first period income is drawn from an endowment and the second period income from past savings in period 1 and wage income in period 2. In particular, a household  $i$  maximises the lifetime (linear) utility,

$$\theta_i c_{1i} + \beta c_{2i}, \quad (1)$$

subject to the following budget constraints and borrowing constraint:

$$c_{1i} = H - s_i \quad (2)$$

$$c_{2i} = s_i R_b + W + D \quad (3)$$

$$s_i \geq -\bar{B}, \quad (4)$$

where  $c_{1i} \geq 0$  is consumption in period 1,  $c_{2i} \geq 0$  is consumption in period 2,  $H$  is endowment in period 1,  $R_b$  is the gross interest rate on savings,  $W$  is wage income in period 2,  $D$  is profit income from firms in period 2, and  $\bar{B} \geq 0$  is the maximum amount a household can borrow in period 1. Also,  $\theta_i$  is a random preference shock drawn independently and identically from a common distribution function. For simplicity, assume that  $\theta_i$  takes the value of 1 with probability  $\pi$  and the value of  $\tau < 1$  with probability  $1 - \pi$ .

**Firms.** A representative firm produces output to meet households' consumption demand in period 1. Its problem is to choose capital ( $K$ ) and labor ( $N$ ) to solve

$$\Pi \equiv \max_{\{K, N\}} \{AK^\alpha N^{1-\alpha} - WN - R_b K\}, \quad (5)$$

subject to the borrowing constraint

$$K \leq \bar{K}. \quad (6)$$

The firm's problem can be simplified by substituting out its optimal labor choice  $N$  in the profit function. Given  $K$  and the real wage  $W$ , the first-order condition of labor choice implies the

following optimal labor demand:

$$N = [A(1 - \alpha)/W]^{\frac{1}{\alpha}} K. \quad (7)$$

Hence, we have  $AK^\alpha N^{1-\alpha} - WN = \alpha A [A(1 - \alpha)/W]^{\frac{1-\alpha}{\alpha}} K \equiv R_k K$ . The firm's problem then becomes  $\Pi = \max_K (R_k - R_b)K$  subject to the constraint (6). Thus, the firm's profit is simply the difference between the rate of return to capital ( $R_k K$ ) and its borrowing costs ( $R_b K$ ). We normalise the equilibrium optimal capital stock to one in the absence of financial frictions (borrowing constraints) by assuming  $\alpha A = 1/\beta$ . We make two additional assumptions:  $(1 - \pi)H > 1 + \pi\bar{B}$  and  $\bar{K} < 1$ , to ensure that the borrowing constraint of the households (4) will bind if  $\theta_i = 1$  and that for the firm (6) will bind in equilibrium.

**Equilibrium in the Closed Economy.** With the labor market clearing condition  $N = 1$ , equation (7) implies  $W = (1 - \alpha)AK^\alpha$ . It then follows that  $R_k = \alpha AK^{\alpha-1}$ . In other words, the real wage is the marginal product of labor and the rate of return to capital equals the marginal product of capital.<sup>17</sup> Hence, in equilibrium we have  $R_b = \tau/\beta < 1/\beta$  and  $R_k = \bar{K}^{\alpha-1}/\beta > 1/\beta$ ,

$$s_i = \begin{cases} -\bar{B} & \text{if } \theta_i = 1, \\ (\bar{K} + \pi\bar{B})/(1 - \pi) & \text{if } \theta_i = \tau; \end{cases} \quad (8)$$

and consumptions  $c_{1i}$  and  $c_{2i}$  are given by (2) and (3), respectively, and the equilibrium capital stock of the firm is  $K = \bar{K}$ .<sup>18</sup>

To gain a better understanding of the distortions in asset returns caused by borrowing constraints, we now characterise the equilibrium without financial frictions: The equilibrium without borrowing constraints (4) and (6) satisfies  $R_b = R_k = 1/\beta$ ,  $K_j = K = 1$ , and  $s_i = H$  if  $\theta_i = \tau$  and  $s_i = [1 - (1 - \pi)H]/\pi$  if  $\theta_i = 1$ . It is easy to verify these equilibrium conditions. First, notice that without borrowing constraint (6), competition for loanable funds will then drive  $R_k$  to equalise  $R_b$ . Second,  $R_b$  must be equal to  $1/\beta$  in equilibrium, because no households would be willing to lend (borrow) if  $R_b < 1/\beta$  ( $R_b > 1/\beta$ ).

Comparing the two equilibria with and without financial frictions, we learn that borrowing constraints reduce the rate of return to financial assets (savings) but increase the rate of return to physical capital for the home country. Thus, if the home country is financially integrated with

<sup>17</sup>The market clearing conditions are:  $\int_0^1 s_i di = K$ ,  $\int_0^1 c_{1i} di + K = H$ ,  $\int_0^1 N_i = N = 1$ , and  $\int_0^1 c_{2i} dj = AK^\alpha N^{1-\alpha}$ .

<sup>18</sup>The proof is straightforward. First given  $R_k > R_b$ , the firm wants to borrow as much as possible to invest, so the constraint (6) binds:  $K = \bar{K}$ . Since  $R_b = \tau/\beta < 1/\beta$ , the impatient households with  $\theta_i = 1$  opt to borrow as much as possible, so  $s_i = -\bar{B}$ . For the patient households with  $\theta_i = \tau$  their saving is determined by the aggregate capital (bond) market clearing condition  $\int_{\theta_i=\tau} s_i di = \bar{K} + \pi\bar{B}$ , where the left-hand side is total savings of the patient households with  $\theta_i = \tau$ , and the right-hand side is total borrowing by the firm and by the impatient households (with  $\theta_i = 1$ ). This is the case because households can only borrow from each other in a closed economy. By imposing a symmetry equilibrium, we obtain the second line in equation (8). Notice by assumption  $(\bar{K} + \pi\bar{B})/(1 - \pi) < H$ , consumption of the impatient in the first period is strictly positive.

a foreign country with no or less financial frictions, financial capital will flow from home (South) to abroad (North) because of a higher interest rate abroad, whereas fixed capital will flow in the opposite direction because of a higher capital return at home. We now proceed to characterise the open-economy equilibrium when the two countries are financially integrated.

**Equilibrium in the Open Economy.**<sup>19</sup> In the open-economy equilibrium, under arbitrage we must have  $R_b^f = 1/\beta = R_k^f$ ,  $R_b = 1/\beta$  and  $R_k = 1/\beta + \varphi$  for any transaction cost of FDI  $\varphi < 1/\beta \min(1 - \tau, \bar{K}^{\alpha-1} - 1)$ . The intuition is as follows. First, since there are no borrowing constraints in the foreign country, we must have  $R_b^f = 1/\beta = R_k^f$ . Second, for the households in the home country, the domestic bond return is  $R_b$  while the foreign bond return is  $1/\beta$ . Since the two bonds are perfect substitutes, arbitrage implies  $R_b = 1/\beta$ . Finally, from the foreign firms' point of view, one unit of domestic investment yields  $R_k^f = 1/\beta$ , and one unit of investment abroad yields  $R_k - \varphi$ . This implies in that equilibrium we must have  $R_k - \varphi = R_k^f = 1/\beta$ , or  $R_k = 1/\beta + \varphi$ . Notice that the local households will hold foreign bonds because  $R_b^f > R_b$  in autarky. Similarly, since  $R_k = 1/\beta + \varphi > R_k^f$ , the local firm will not invest in the foreign country. Because  $R_k > R_b$ , the borrowing constraint (6) will be binding,  $K_j = \bar{K}$ , in the home country.<sup>20</sup> The amount of physical capital that is imported from the foreign country is  $(1 + \beta\varphi)^{\frac{1}{\alpha-1}} - \bar{K} > 0$ , and the amount of financial capital going to the foreign country is given by  $(1 - \pi)H - \bar{K} - \pi\bar{B} > 0$ . Notice that the transaction cost  $\varphi$  determines the size of physical capital flows.

**Firm Heterogeneity.** The toy model illustrates that financial frictions alone can in principle explain the two-way capital flows between developing countries and developed countries. However the simple toy model has a counterfactual implication about the aggregate domestic saving rates in the two countries. The saving rate in the home country is  $\bar{K}/H$ , while the saving rate in the foreign country is  $1/H$ . Since by assumption  $\bar{K} < 1$ , we have  $\bar{K}/H < 1/H$ . However, in the data developing countries tend to have a higher aggregate saving rate than developed countries—the so-called "savings glut". To solve this problem, we will introduce firm heterogeneity into the following full-fledged model.

<sup>19</sup>In the open economy, households in each country can save in both domestic bonds and foreign bonds, and firms can invest both at home and abroad by shipping physical capital across borders. When the firm sends capital abroad to produce output, it hires foreign workers and pays foreign wages. We assume that there is a transaction cost  $\varphi$  in shipping physical capital abroad. So the home country household budget constraints change to  $c_{i1} = H - s_i - \tilde{s}_i$  and  $c_{i2} = s_i R_b + W + D + \tilde{s}_i R_b^f$ , where  $\tilde{s}_i \geq 0$  is the foreign bond holdings,  $R_b^f$  denotes the foreign interest rate. The borrowing constraint changes to  $s_i + \tilde{s}_i \geq -\bar{B}$ . The home country firm can invest  $u_j \in [0, 1]$  fraction of its capital in the foreign country. The firm's problem becomes  $\max_{\{u, N, X, K\}} \{[(1-u)K]^\alpha N^{1-\alpha} - W^h N + (uK)^\alpha X^{1-\alpha} - W^f X - \varphi uK - R_b K\}$ , where  $W^f$  denotes foreign wage and  $X_j$  foreign labor. The borrowing constraint of the firm is still given by equation (6). The households and firm in the foreign country solve similar problems except they do not face borrowing constraints.

<sup>20</sup>Also note, since  $\tau/\beta < R_b$ , we must have  $s_i + \tilde{s}_i = \begin{cases} -\bar{B} & \text{if } \theta_i = 1, \\ H & \text{if } \theta_i = \tau. \end{cases}$

### 3 The Full-Fledged Dynamic Model

The full-fledged dynamic two-country model is an infinite horizon model with standard preferences and production technologies. The model yields the same qualitative predictions for two-way capital flows as the toy model, but is more sophisticated to permit calibrations and quantitative studies. As in the toy model, the two countries are indicated by  $h$  and  $f$ . There are two types heterogeneous agents in both countries. We use  $i \in [0, 1]$  to index heterogeneous households and  $j \in [0, 1]$  to index heterogeneous firms. Each country issues its own country-specific bonds, and neither country can issue foreign bonds. To simplify the analysis we assume that bonds are the only tradable financial assets between the two countries.<sup>21</sup> However, firms can invest in the foreign country through FDI. We use the tightness of borrowing constraints to indicate the degree of financial development in each country, as is standard in the literature (e.g., MQR, 2009). Because firms are heterogeneous, each consumer holds a portfolio of firms' equities, taking as given the market prices of the portfolio.

We focus our analysis on the home country in what follows. The foreign country's problem is analogous. Whenever convenient, we use  $\ell \in \{h, f\}$  as the country index and use  $\ell_c$  to denote the counterpart of country  $\ell$ .

#### 3.1 Households

In each period  $t$ , household  $i$  derives utility from consumption  $c_{it}$  and leisure  $1 - n_{it}$ . The instantaneous utility function is quasi-linear,  $\theta_{it} \log c_{it} - \psi n_{it}$ , where the preference shock  $\theta_{it}$  is drawn from a common distribution  $F(\theta) = \Pr[\theta_i \leq \theta]$  with support  $[\theta_{\min}, \theta_{\max}]$ . Each period is divided into two subperiods. The idiosyncratic preference shocks are realised in the second subperiod. Each household  $i$  chooses labor supply  $n_{it}$  in the first subperiod without observing  $\theta_{it}$ . This implies that households cannot use the labor supply to insure themselves against the idiosyncratic shocks. Consumption and saving decisions are made in the second subperiod after preference shocks are realised. Specifically, after choosing  $n_{it}$  and upon observing  $\theta_{it}$ , household  $i$  chooses consumption  $c_{it}$ , savings in domestic bonds  $s_{it+1}$ , savings in foreign bonds  $\tilde{s}_{it+1}$ , and savings in firms' equities  $a_{it+1}$ . As shown by Wen (2009, 2015), such an information structure permits closed-form solutions for household decision rules with incomplete markets and borrowing constraints.

Denoting  $Q_t$  as the price index of a portfolio of firms' equities (stocks) and  $D_t$  as the aggregate dividend paid to the portfolio (capturing the rate of return to stocks), the borrowing constraint facing each household is specified as

$$s_{it+1} + \tilde{s}_{it+1} + a_{it+1}Q_t \geq -B_t, \quad (9)$$

<sup>21</sup>Allowing households to hold foreign firms' equities does not change our results qualitatively. This simplifying assumption is made so we can focus on FDI in the form of shipping fixed capital across borders and not mangle it with acquiring the ownership of foreign firms through equity holdings.

where  $a_{it+1}$  is the share of the portfolio newly purchased by the household in period  $t$ , and  $B_t \geq 0$  is an exogenously specified borrowing limit (as in Aiyagari, 1994). To facilitate analysis, we assume that  $B_t$  is proportional to the value of equity,  $B_t = bQ_t$ , where  $b$  captures the degree of financial development on the household side.

Since countries cannot issue foreign bonds (although households can hold foreign bonds), we have

$$\tilde{s}_{it+1} \geq 0, \quad (10)$$

for all  $i \in [0, 1]$ . This implies that if a country opts to borrow abroad, it must sell its home bonds to foreigners.<sup>22</sup>

Taking as given the real wage  $W_t$  and the real interest rates at home and abroad, household  $i$  solves

$$\max_{\{n_{it}, c_{it}, s_{it+1}, \tilde{s}_{it+1}, a_{it+1}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\theta_{it} \log c_{it} - \psi n_{it}) \right], \quad (11)$$

subject to constraints (9) and (10), as well as the budget constraint

$$c_{it} + s_{it+1} + \tilde{s}_{it+1} + a_{it+1}Q_t \leq R_{bt-1}^h s_{it} + R_{bt-1}^f \tilde{s}_{it} - \gamma_s \tilde{s}_{it}^{1+\tau} / (1 + \tau) + W_t n_{it} + (Q_t + D_t) a_{it}, \quad (12)$$

where  $\{R_{bt}^h, R_{bt}^f\}$  denote domestic and foreign interest rates, respectively, and  $\gamma_s \tilde{s}_{it}^{1+\tau} / (1 + \tau)$  denotes the convex cross-border trading costs for purchasing foreign bonds (with  $\gamma_s \geq 0$  and  $\tau > 0$ ).<sup>23</sup>

### 3.2 Firms

Each firm  $j$  with capital stock  $K_{jt}$  can choose to produce both at home and abroad. A firm combines labor and capital to produce output through the Cobb-Douglas technology  $Y_{jt} = K_{jt}^\alpha N_{jt}^{1-\alpha}$ . Each firm accumulates productive capital according to the law of motion,

$$K_{jt+1} = (1 - \delta) K_{jt} + \varepsilon_{jt} I_{jt}, \quad (13)$$

where  $I_{jt}$  denotes investment expenditures and  $\varepsilon_{jt} \in \mathbb{R}^+$  is an idiosyncratic shock to the marginal efficiency of investment, which is i.i.d across firms and over time (as in Wang and Wen, 2012). We denote the cumulative density function of  $\varepsilon$  by  $\Phi(\varepsilon)$ .

<sup>22</sup>The constraint in equation (10) is not essential. Our general results hold if we simply allow an international bond with a world interest rate. However, to capture the different interest rates in China and the U.S. both before and after financial liberalisation, we need to have domestic and foreign bonds with asymmetric trading costs.

<sup>23</sup>We assume that there exist cross-border trading costs in purchasing foreign bonds and the costs are increasing in the trading volume. This assumption is not necessary for our general results but is needed only to capture the transitional dynamics of international financial capital flows after financial liberalisation. China opens its capital markets only gradually; even today its capital markets are not completely open. So the rationale for such trading costs includes capital controls in developing countries in addition to other transaction costs discussed in the literature. However, our qualitative results do not hinge on the assumption of trading costs and our model nests the standard models with zero trading costs as a special case.

With heterogeneous households, the firm's dynamic programming problem becomes slightly more complicated. The first step is to find the correct discount factor. We follow Hansen and Richard (1987) and Cochrane (1991) to assume that there exists a sequence of prices  $\{P_t\}_{t=0}^{\infty}$  such that a firm's expected value is determined by

$$V_{jt} = E_t \sum_{\tau=0}^{\infty} (P_{t+\tau}/P_t) D_{jt+\tau}, \quad (14)$$

where  $\{D_{jt+\tau}\}_{\tau=0}^{\infty}$  is the dividend flows generated by firm  $j$  and the expectation operator  $E$  is taken on the idiosyncratic shock  $\varepsilon_{jt}$ . Denoting  $\Lambda_t \equiv P_t/\rho^t$ , where  $\rho < 1$ , we can rewrite the firm's expected value as  $V_{jt} = E_t \sum_{\tau=0}^{\infty} \rho^{\tau} (\Lambda_{t+\tau}/\Lambda_t) D_{jt+\tau}$ , which can be rewritten recursively as<sup>24</sup>

$$V_{jt} = \int [D_{jt} + \rho E_t (\Lambda_{t+1}/\Lambda_t) V_{jt+1}] d\Phi. \quad (15)$$

Notice that because of heterogeneity on the household side,  $\rho$  does not necessarily equal the household's discount factor  $\beta$ . With the firm value given by equation (15), the firm's problem is then to maximise its expected value  $V_{jt}$  by choosing labor demand, capital allocation (the share of FDI), and the level of fixed investment.

All firms' decisions are made after observing their idiosyncratic shock  $\varepsilon_{jt}$  in the beginning of each period. Specifically, firm  $j$  decides to allocate  $1 - u_{jt}$  fraction of its fixed capital stock ( $K_{jt}$ ) at home and the remaining  $u_{jt}$  fraction of the capital stock abroad.<sup>25</sup> We assume that there are costs involved in reallocating fixed capital across borders and a firm needs to pay the amount  $\gamma_k u_{jt}^{1+\chi} / (1 + \chi) K_{jt}$  to move  $u_{jt}$  fraction of its capital stock abroad. This cost is analogous to the transaction cost  $\varphi$  in the toy model.<sup>26</sup> The parameters  $\gamma_k (> 0)$  and  $\chi (> 0)$  control capital mobility and the extent of openness for the fixed capital market. For example, when  $\gamma_k = \infty$ , cross-border fixed capital flows are completely shut down. When  $\gamma_k = 0$ , FDI flows can be adjusted instantaneously without any costs. This parameter also captures institutional costs for setting up foreign business and policies designed to attract FDI through reducing such frictions.

The optimal choices of  $u_{jt}$  as well as labor inputs are static. Given the capital stock  $K_{jt}$ , firm

<sup>24</sup>Notice that by our definition of firm's value, the value function  $V_{jt}$  is independent of the firm's idiosyncratic shock  $\varepsilon_{jt}$  in period  $t$ . This approach simplifies our notation but is not essential for our results. Alternatively, we could define a firm's value as  $V_{jt} = D_{jt} + \rho E_t (\Lambda_{t+1}/\Lambda_t) V_{jt+1}$ , so that it depends on period- $t$ 's shock  $\varepsilon_{jt}$ .

<sup>25</sup>The outward FDI of the home country in our model is thus  $u_t K_t$ . According to the BEA's data, this form of FDI dominates other forms of FDI flows in U.S. economy.

<sup>26</sup>Even though financial and fixed capital move in opposite directions, net aggregate capital (financial plus fixed) still shows the reversed pattern noticed by Lucas (1990). The net foreign asset position of a country (the sum of net flows in financial and fixed capitals) is determined by the liquidity (mobility) of the two forms of capital. Thus the transaction costs allow our model to quantitatively match the imbalanced two-way capital flows between China and the U.S.

$j$ 's operating profits  $\Pi_{jt}$  can be derived through the following maximisation problem:

$$\Pi_{jt} \equiv \max_{\{u_{jt}, N_{jt}, X_{jt}\}} \left\{ [(1 - u_{jt})K_{jt}]^\alpha N_{jt}^{1-\alpha} - W_t^h N_{jt} + (u_{jt}K_{jt})^\alpha X_{jt}^{1-\alpha} - W_t^f X_{jt} - \gamma_k u_{jt}^{1+\chi} / (1 + \chi) K_{jt} \right\} \quad (16)$$

where  $W_t^h$  and  $W_t^f$  are the real wage in the home country and the foreign country, respectively,  $N_{jt}$  the demand for domestic labor, and  $X_{jt}$  the demand for foreign labor. Let  $r_t$  denote the marginal product of domestic capital ( $\partial Y_{jt} / \partial [(1 - u_{jt})K_{jt}]$ ) and  $R_{kt}$  the gross marginal product of capital (including capital operating both at home and abroad). Appendix A.1 shows that  $r_t$  and  $R_{kt}$  are both independent of firm-specific shocks and are related by the following relationship:

$$R_{kt} = r_t + \mathbf{1}_{r_t^f > r_t} \left[ \chi / (1 + \chi) \gamma_k^{-\frac{1}{\chi}} \left( r_t^f - r_t \right)^{\frac{1+\chi}{\chi}} \right], \quad (17)$$

where  $\mathbf{1}_{r_t^f > r_t}$  is an index function that takes value of 1 whenever  $r_t^f > r_t$  and 0 otherwise. We now define the MPK as the gross marginal product of capital net of depreciation rate:

$$MPK_t \equiv R_{kt} - \delta. \quad (18)$$

Notice that in financial autarky (no cross-border capital flows),  $MPK_t = r_t - \delta$ , which is the conventional definition of the MPK used in the empirical literature (e.g., Bai *et al.*, 2006).

We now discuss the firm's dynamic optimisation problem in choosing investment  $I_{jt}$  (after observing  $\varepsilon_{jt}$ ). Let  $V_t(K_{jt})$  denote the expected value of the firm with capital stock  $K_{jt}$  at the beginning of period  $t$  before observing  $\varepsilon_{jt}$ . This value function can now be defined recursively using the proper discount factor  $\rho\Lambda_{t+1}/\Lambda_t$  as

$$V_t(K_{jt}) = \int \max_{I_{jt}} [\Pi_{jt} - I_{jt} + E_t(\rho\Lambda_{t+1}/\Lambda_t) V_{t+1}((1 - \delta)K_{jt} + \varepsilon_{jt}I_{jt})] d\Phi, \quad (19)$$

where  $\Pi_{jt} - I_{jt} \equiv D_{jt}$  is dividend. We assume that a firm can use both internal funds,  $\Pi_{jt}(K_{jt})$ , and outside funds (from borrowing),  $L_{jt}$ , to finance investment. Hence, the maximum investment is subject to the constraint

$$I_{jt} \leq L_{jt} + \Pi_{jt}. \quad (20)$$

For simplicity, we assume that the external funds are raised through intraperiod loans. Firms can borrow from each other through a financial intermediary at the beginning of period  $t$  and pay back the loan at the end of period  $t$  with zero interest rate.<sup>27</sup> Since in each period some firms

<sup>27</sup>Because of irreversible investment and the option value of waiting, unproductive firms with low  $\varepsilon$  shock opt not to invest and prefer saving through the financial intermediary (as a form of liquidity). The zero interest rate is an innocuous assumption. Allowing for a one-period loan with positive interest rate does not change our results. This is equivalent to a one-period private bond market where firms lend and borrow from each other by issuing (purchasing) private bonds (the formal proof is available upon request). Since we already have a interest rate on government bonds, eliminating the interest rate on private bonds can simplify our analysis and notations.

will opt not to invest ( $I_{jt} = 0$ ), financial intermediaries can lend these inactive firms' savings to investing firms after paying dividends to equity holders (households).

Loans are subject to collateral constraints, as in Kiyotaki and Moore (1997). That is, firm  $j$  is allowed to pledge a fraction  $\xi \in (0, 1]$  of its fixed capital stock  $K_{jt}$  at the beginning of period  $t$  as collateral. In general, the parameter  $\xi$  represents the extent of financial market imperfections—the higher the value of  $\xi$ , the more a firm can borrow and thus the more advanced the financial market. At the end of period  $t$ , the market value of the pledged collateral is equal to  $\rho(\Lambda_{t+1}/\Lambda_t) V_{t+1}(\xi K_{jt})$ , which is the present value of the collateral of firm  $j$  at the beginning of period  $t + 1$ , or equivalently the value of a firm that owns collateralisable capital stock  $\xi K_{jt}$ . The amount of loans  $L_{jt}$  cannot exceed this collateral value because of limited contract enforcement. Thus, we impose the following collateral constraint:

$$L_{jt} \leq \rho(\Lambda_{t+1}/\Lambda_t) V_{t+1}(\xi K_{jt}). \quad (21)$$

We also assume that investment is irreversible,

$$I_{jt} \geq 0. \quad (22)$$

To summarise, each firm  $j$  solves the static problem (16) and the dynamic programming problem (19) subject to constraints (20), (21), and (22).

The sequence of events and information structure can be summarised as follows:

1. The households make labor supply decisions without knowing their idiosyncratic preference shocks  $\theta_t(i)$ .
2.  $\theta_t(i)$  and  $\varepsilon_t(i)$  are realised. The households make consumption and saving decisions based on  $\theta_t(i)$ . Firms make their hiring, investment, borrowing/lending, and dividend payment decisions based on  $\varepsilon_t(i)$ .

### 3.3 *Financial Intermediation*

The financial intermediation in our model is stylised. A representative financial intermediary holds a portfolio consisting of all firms' stocks and collects the aggregate dividends  $D_t$  from all firms, i.e.,  $D_t = \int D_{jt} dj$ . Although the financial intermediary makes intra-period loans to firms using the dividends, the loans are all repaid within the period, so they do not affect the end-of-period aggregate dividends. The price of such portfolio,  $Q_t$ , is hence

$$Q_t = \rho(\Lambda_{t+1}/\Lambda_t) (Q_{t+1} + D_{t+1}). \quad (23)$$

Alternatively, we can also assume that households themselves hold a market portfolio consisting of the stocks of all firms and the equilibrium results will be the same.



### 3.4 General Equilibrium

We denote the aggregate capital stock, aggregate investment, aggregate labor demand, aggregate output, aggregate labor supply, aggregate bond holdings, aggregate household savings, and aggregate consumption in country  $\ell$  by  $K_t^\ell = \int_0^1 K_{jt}^\ell dj$ ,  $I_t^\ell = \int_0^1 I_{jt}^\ell dj$ ,  $N_t^\ell = \int_0^1 N_{jt}^\ell dj$ ,  $X_t^\ell = \int_0^1 X_{jt}^\ell dj$ ,  $Y_t^\ell = \int_0^1 Y_{jt}^\ell dj$ ,  $n_t^\ell = \int_0^1 n_{it}^\ell di$ ,  $S_t^\ell = \int s_{it}^\ell di$ ,  $\tilde{S}_t^\ell = \int \tilde{s}_{it}^\ell di$ , and  $C_t^\ell = \int_0^1 c_{it}^\ell di$ , respectively. The general equilibrium of the model is defined as the sequences of aggregate variables,  $\{K_t^\ell, I_t^\ell, N_t^\ell, X_t^\ell, Y_t^\ell, n_t^\ell, S_t^\ell, \tilde{S}_t^\ell, C_t^\ell\}$ , individual firms' decisions,  $\{K_{jt}^\ell, I_{jt}^\ell, N_{jt}^\ell, L_{jt}^\ell, Y_{jt}^\ell\}$ , individual households' choices,  $\{a_{it}^\ell, n_{it}^\ell, s_{it}^\ell, \tilde{s}_{it}^\ell, c_{it}^\ell\}$ , and aggregate prices,  $\{Q_t^\ell, W_t^\ell, R_{kt}^\ell, R_{bt}^\ell\}$ , for  $\ell \in \{h, f\}$ , such that each firm or each household solves its optimisation problem and all markets (labor, equity, and bonds markets) clear:

$$N_t^\ell + X_t^{\ell c} \equiv \int_0^1 N_{jt}^\ell dj + \int_0^1 X_{jt}^{\ell c} dj = n_t^\ell \quad (24)$$

$$\int a_{it}^\ell di = 1. \quad (25)$$

Notice that in a financial autarky regime, the bond market-clearing condition is  $S_t^\ell = \tilde{S}_t^\ell = 0$ , whereas in a financial liberalisation regime, the bond market-clearing condition is

$$S_t^\ell + \tilde{S}_t^{\ell c} = 0, \quad (26)$$

where  $\tilde{S}_t^\ell$  denotes country  $\ell$ 's holdings of the other country's bonds. The aggregate capital stock evolves according to

$$K_{t+1}^\ell = (1 - \delta) K_t^\ell + \int \varepsilon_{jt}^\ell I_{jt}^\ell dj. \quad (27)$$

### 3.5 Solving the General Equilibrium

Since our model has closed-form solutions for decision rules of both households and firms and the equilibrium distributions of households and firms can be fully characterised by two cutoff variables in each country (with the cutoff variables depending only on aggregate states but not on individual histories, see the discussions in Appendix A), the general equilibrium of our model can be solved easily in following three steps. (i) Solve the decision rules of individual households and firms. (ii) Aggregate the individual decision rules under the law of large numbers to form a system of dynamic nonlinear equations expressed in the aggregate variables. And (iii) solve the aggregate policy rules from the nonlinear system of aggregate equations by the standard numerical method

used in the literature. To conserve space, we characterise the decision rules and the system of non-linear equations in several propositions in Appendix A and the proofs in online Appendix B.

In Appendix A.3 we also show that the aggregate economy exhibits two wedges, a saving wedge and an investment wedge, which create the driving forces of two-way capital flows. The saving wedge is related to the "aggregate" household's intertemporal Euler equation of consumption and saving, and the investment wedge pertains to the "aggregate" firm's investment decisions or Tobin's  $q$ . The first wedge generates an excessively low interest rate due to borrowing constraints on the household side that create a motive for precautionary saving and a liquidity premium on bond returns, which push down the financial interest rate on household savings. The second wedge generates an excessively high MPK due to borrowing constraints on the firm side that raise the equilibrium Tobin's  $q$ . Even though the MPK is very high, firms cannot invest enough because of the borrowing constraints.

## 4 International Capital Flows

Everything else being equal, the directions of international capital flows depend on the differential interest rates and MPKs across countries, which in turn depend on the demand and supply of capital and the degree of financial development in each country. This section characterises the relationships among the borrowing constraint parameters  $\{b^\ell, \xi^\ell\}$ , the interest rates  $\{R_b^\ell\}$ , and the MPKs  $\{MPK_t^\ell\}$  for  $\ell \in \{h, f\}$  through the lens of demand and supply of capital in each country and explains how they interact to determine the equilibrium interest rate and the MPK.

In the model, both households and firms can save. Households save through bonds and equities (financial assets), whereas firms save through a domestic intra-period loan market (i.e., a corporate union) with participation only from domestic firms. Firms will invest if and only if they find good investment opportunities and will save (remain inactive) otherwise. Because it is costless for firms to borrow from the corporate union (i.e., they pay zero interest for loans), only household savings depend directly on the interest rate in the financial market. In particular, the aggregate household savings depend positively on the interest rate.

Firms' investments are financed by two sources: internal cash flows and outside credit from the corporate union. Borrowing from the corporate union is free but subject to borrowing constraints.<sup>28</sup> Hence, the aggregate demand for capital depends indirectly on the financial interest rate through the rate of return to equities. When the interest rate is high, the rate of return to equities must also be high to attract equity buyers. This means that either the equity price must be low or the

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<sup>28</sup>Our results do not change if firms must pay interest rate to the corporate union. But this will complicate the problem because we then have two interest rates, one for the household and another for the firms. Also, having an additional interest rate in the private bond market will enhance our results because it will further increase firms' MPK, everything else being equal.

dividend payment must be high. In either case, the present value of a firm's internal cash flows is reduced, which will decrease a firm's investment demand. In addition to this intensive margin, a reduced equity price also raises the threshold (cutoff) of investing, thus lowering the aggregate investment through the extensive margin. Therefore, the aggregate demand for capital depends negatively on the interest rate, among other things.

Thus household savings are channeled to firms only through the equity market and they affect firms' investment demand through equity prices and dividends. When the household saving rate is high, the demand for equities will increase. In equilibrium, either the equity price level will increase or the average dividends will decrease; in either case, the rate of return to equities must decline. By arbitrage, the interest rate on bonds must also decline. This has positive effects on firms' investment demand because (i) a lower interest rate increases firms' present value of future cash flows due to a lower discount rate, and (ii) a lower dividend payment improves firms' cash positions. A higher rate of investment from firms will then reduce the MPK. This suggests that financial capital inflows from other countries can lower the domestic interest rate and the MPK of the home country. On the other hand, fixed capital inflows from foreign countries will (i) reduce the MPK at home and (ii) lower the domestic interest rate because it reduces the equity return at home.

For the U.S., the inflows of financial capital will decrease the domestic interest rate and the MPK, but meanwhile the outflows of FDI will increase its interest rate and the MPK. Therefore, two-way capital flows have the opposite effects on domestic interest rate and the MPK. This suggests that financial liberalisation may not necessarily decrease the U.S. interest rate unless financial capital inflows dominate FDI outflows. On the other hand, the effects of financial development on the interest rate and the MPK are somewhat different from those of capital flows and are more complicated because changes in the borrowing constraints (e.g.,  $\xi$ ) have ambiguous effects on the interest rate (since they simultaneously shift the demand and supply curves of capital). These effects are studied next.

#### 4.1 *Equilibrium in the Capital Market*

Recall that the market clearing condition for international bonds determines foreign reserves  $\tilde{S}^h$  at home and  $\tilde{S}^f$  abroad. From equation (26), we have

$$\tilde{S}^h = -S^f \text{ and } S^h = -\tilde{S}^f. \quad (28)$$

The general equilibrium of the two-country model under financial integration can be characterised by the equilibrium capital-to-output ratios  $\{\bar{K}^h/Y^h, \bar{K}^f/Y^f\}$  and the real interest rates  $\{R_b^h, R_b^f\}$ ,

which are determined jointly by the demand and supply of capital in the two countries (see Appendix A.4).<sup>29</sup>

However, to understand the factors determining the rates of return to capital and the directions of capital flows, it helps to study a world without any form of international capital flows—the financial autarky regime. To obtain a financial autarky regime, we can simply set the cost parameters  $\gamma_k$  and  $\gamma_s$  to infinity so that there are no cross-border flows of financial and fixed capital (i.e.,  $u^\ell = S^\ell = \tilde{S}^\ell = 0$ ). In a financial autarky equilibrium, the demand function (A.30) and supply function (A.32) of capital in Appendix A.4 for the two countries collapse to

$$\alpha Y^\ell / K^\ell = \mathbb{R} \left( R_b^\ell, \xi^\ell \right), \quad (29)$$

$$(1 - \alpha) Y^\ell / K^\ell = \left[ \Upsilon \left( R_b^\ell \right) - R_b^\ell + 1 \right] \mathbb{Q} \left( R_b^\ell, \xi^\ell \right) + \Upsilon \left( R_b^\ell \right) \mathbb{Q} \left( R_b^\ell, \xi^\ell \right) b^\ell, \quad (30)$$

where  $\partial \mathbb{R} \left( R_b^\ell, \xi^\ell \right) / \partial R_b^\ell > 0$ ,  $\partial \mathbb{R} \left( R_b^\ell, \xi^\ell \right) / \partial \xi^\ell < 0$ ,  $\Upsilon' \left( R_b^\ell \right) < 0$ ,  $\partial \mathbb{Q} \left( R_b^\ell, \xi^\ell \right) / \partial R_b^\ell < 0$  and  $\partial \mathbb{Q} \left( R_b^\ell, \xi^\ell \right) / \partial \xi^\ell$ , for  $\ell = \{h, f\}$ . Note that due to the immobility of both fixed and financial capital, there is no interaction between the two countries, and the equilibrium capital-to-output ratio and interest rate in each country are then fully pinned down by the domestic capital demand curve and domestic capital supply curve in equations (29) and (30).

**Proposition 1** *In the financial autarky regime, the country with tighter borrowing constraints on the firm side (i.e., smaller  $\xi$ ) has a higher MPK but either a higher or a lower domestic interest rate; and the country with tighter borrowing constraints on the household side (i.e., smaller  $b$ ) has both a lower MPK and a lower real interest rate.*

The proof is straightforward and is illustrated graphically in Figure 5. The left panel is the autarky equilibrium in which the two countries differ only in the tightness of borrowing constraints on the firm side, but they have the same tightness of borrowing constraints on the household side. Suppose firms in country  $f$  can borrow more than firms in country  $h$ :  $\xi^f > \xi^h$ . The "S-S" curve represents capital supply and the "D-D" curve capital demand, and point  $H$  represents autarky equilibrium in country  $h$  and point  $F$  autarky equilibrium in country  $f$ . According to equations (29) and (30), a larger  $\xi$  will shift both the demand and the supply curves towards the right. As a result, point  $H$  will lie to the left side of point  $F$  and the home country will have a lower capital-to-output ratio (or a higher MPK). The rank of interest rates in the two countries, however, is ambiguous since point  $F$  can be either above or below point  $H$ , depending on the magnitudes of the right-ward shifts of the two curves. The intuition is that looser borrowing constraints on firms in

<sup>29</sup>  $\bar{K}^\ell$  denotes total capital (domestic and foreign) utilised in production in country  $\ell$ , which includes both domestic capital and FDI.

the foreign country lead to a higher demand for capital, which shifts out the "D-D" demand curve directly and results in a lower Tobin's  $q$  due to the lowered MPK. A lower Tobin's  $q$  in turn leads to a lower equity price ( $Q$ ). Thus, households are willing to buy more equities or, equivalently, save more. As a result, the "S-S" supply curve will also shift out to the right. Consequently, whether the equilibrium interest rate is lower or higher than that in country  $h$  is ambiguous.

The right panel in Figure 5 illustrates the case in which the tightness of borrowing constraint is identical on the firm side between the two countries but differs on the household side. Assume households in country  $f$  are less borrowing constrained, i.e.,  $b^f < b^h$ . From equation (29), the two countries have thus identical capital demand since  $\xi^h = \xi^f$ , but the capital supply curve in the foreign country lies to the left of the home country's. This occurs because households in the foreign country tend to borrow more and save less due to a less constrained borrowing limit. In equilibrium, the foreign country (point  $F$ ) ends up with both a higher interest rate and a higher MPK (lower capital-to-output ratio) than the home country (point  $H$ ).

This result shows that LDCs could have both a lower interest rate and a lower MPK than developed countries. Consequently, both financial capital and fixed capital should flow from South to North. Although such a one-way unidirectional capital flow is observed in the real world for some developing countries (such as the oil-exporting countries in the Middle East), it is not the dominant pattern of international capital flows. Hence, explaining the two-way capital flow puzzle requires borrowing constraints on both the household side and the firm side.

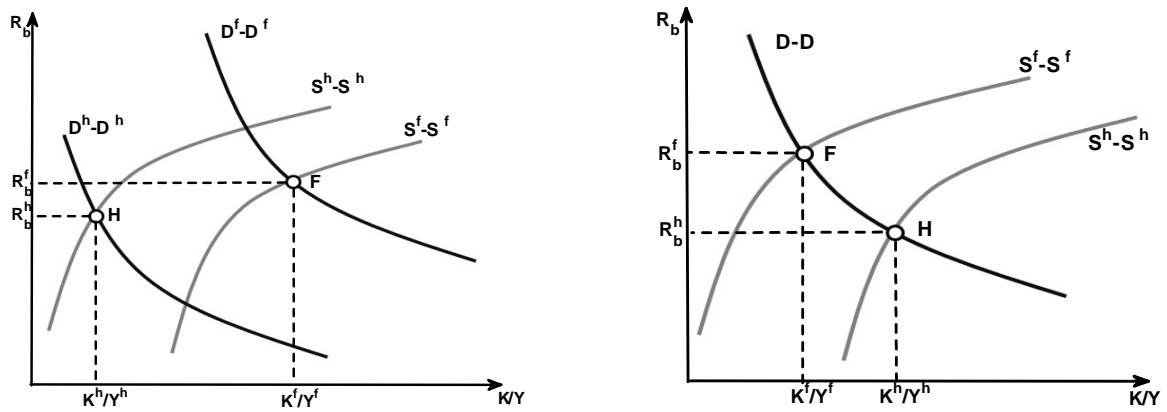


Figure 5. *Steady-State Equilibrium in Financial Autarky*

## 4.2 Two-Way Capital Flows

**Proposition 2** *Moving from financial autarky to financial liberalisation (i.e.,  $\gamma_k < \infty$  and  $\gamma_s < \infty$ ), financial capital will flow from country  $h$  to country  $f$  and fixed capital (FDI) will flow in the*

opposite direction simultaneously if one of the following sets of conditions are satisfied: (i)  $\xi^h < \xi^f$  and  $\underline{b}(\xi^f, \xi^h, b^f) < b^h < \bar{b}(\xi^f, \xi^h, b^f)$ , or (ii)  $b^h < b^f$  and  $\xi^h < \underline{\xi}(\xi^f, b^h, b^f)$ , provided that  $\varepsilon_{jt}$  is Pareto distributed.

**Proof.** See Appendix A.5. ■

As discussed before (see Figure 5), the assumption of  $\xi^h < \xi^f$  alone guarantees that the home country has a higher MPK in autarky and thus it would attract FDI from abroad. However, the direction of financial capital flow is ambiguous in this case because the autarky interest rate at home can be either lower or higher than the foreign interest rate. Therefore, to ensure a lower interest rate at home, the household side must also face a tight enough borrowing constraint ( $b^h < \bar{b}$ ). However, since a tighter borrowing constraint on the household side also lowers the MPK at home, the value of  $b^h$  cannot be too low (i.e.,  $b^h > \underline{b}$ ). This explains the first set of conditions in the proposition.

On the other hand, the assumption of  $b^h < b^f$  alone ensures that the home country has both a lower interest rate and a lower MPK, so we also need a tight enough borrowing constraint on the firm side at home (or a loose enough borrowing constraint abroad) to induce a higher MPK at home than abroad. However, although a lower  $\xi^h$  at home induces a higher MPK, its effect on the interest rate  $R_b^h$  is ambiguous. Therefore, we do not know if the home country will necessarily have a lower interest rate if  $\xi^h$  is reduced. One special case is that if  $\varepsilon$  follows the Pareto distribution, then the interest rate depends only on  $b^\ell$ , so the value of  $\xi^\ell$  does not affect the interest rate. This explains the second set of conditions in the proposition.

These important conditions required in the above proposition to generate the two-way capital flows explain why we do not always observe two-way capital flows between developing and developed countries. For example, among the 22 emerging markets studied in Section 1, two of them do not have negative position in net FDI asset (FDI inflow), and only 15 of them present net financial capital outflow starting from 2000. Also, for the group of 21 developed countries, only 16 of them exhibit the two-way capital flow pattern, although the group as a whole presents very significant two-way pattern—i.e., financial capital inflow and FDI outflow.

FDI and financial capital flows tend to reinforce each other in the opposite directions through their general-equilibrium effects on the interest rate and MPK. Specifically, FDI flows from  $f$  to  $h$  tend to drive out  $h$ 's financial capital because inward FDI lowers the domestic interest rate; and bond flows from  $h$  to  $f$  tend to drive out fixed capital in  $f$  toward  $h$  because inward financial capital flows brings down  $f$ 's MPK. Therefore, the parameter requirements on the values of  $\{\xi^\ell, b^\ell\}$  for triggering two-way capital flows are easier to satisfy than they appear to be in Proposition 2.

### 4.3 Balance of Payments

Persistent net capital inflows would imply a current account deficit in the short run but a trade surplus in the long run because of positive interest payments in the steady state.<sup>30</sup> Since conventional wisdom has it that unidirectional one-way capital flow is not sustainable (i.e., a country's change of net foreign asset position should be zero in steady state), current account should always be balanced in the long run.<sup>31</sup> However, if financial capital (bonds) and fixed capital (FDI) earn different rates of return and they flow in the opposite directions, a country can sustain a long-run trade deficit (or a long-run surplus) even if its net capital (financial and fixed) inflows are balanced at zero and there is no long-run growth. For example, if the U.S. gleans a substantially larger rate of return from foreign capital than foreign investors do from owning U.S. capital (as in the data), it could run substantial trade deficits forever. Conversely, if China holds most of the world's low-yield foreign reserves and pays the highest rate of return to FDI inflows from rich countries, it will experience a trade surplus even in the long run.<sup>32</sup>

The two forms of capital flows have the opposite effects on the domestic interest rate. Therefore, it is not clear *a priori* that financial capital inflows from the South would necessarily reduce the interest rate in the North because FDI outflows will raise it through a higher MPK in the North. In addition, these two forms of capital flows reinforce each other through general equilibrium effects on the interest rate. For example, FDI inflows may crowd out domestic fixed capital investment in the recipient country and push down the real interest rate, which in turn can trigger financial capital outflows. This in turn may restore the interest rate to its original level. On the other hand, financial capital inflows reduce the real interest rate and the MPK in the recipient country, thus causing FDI outflows, which in turn raises the interest rate.

The balance of payments is straightforward to compute in our model. For bond flows we have  $S_t^h = -\tilde{S}_t^f$  and  $\tilde{S}_t^h = -S_t^f$ . Moreover, either  $S_t^h > 0$  or  $\tilde{S}_t^h > 0$ , but not both. For fixed capital flows, only one of the following conditions is true: either  $u_t^h > 0$  or  $u_t^f > 0$ , but not both. Suppose  $\tilde{S}_t^h > 0$  and  $u_t^f > 0$  (as in the data). The current account balance of the home country ( $CA_t^h$ ) in period  $t$  is then given by

$$CA_t^h = \left( \tilde{S}_{t+1}^h - \tilde{S}_t^h \right) - \left( u_t^f K_t^f - u_{t-1}^f K_{t-1}^f \right), \quad (31)$$

where the terms inside the first bracket are the changes in financial asset positions and those in

<sup>30</sup>Following MQR (2009), a nation's current account balance ( $CA_t$ ) is defined as the net changes in foreign asset positions ( $NFA_t$ ):  $CA_t = NFA_t - NFA_{t-1}$ , which is zero in the steady state. Since the current account equals net exports ( $NX_t$ ) plus net factor payments ( $r_t NFA_t$ ), where  $r$  denotes the rate of return, we have in the steady state  $NX = -rNFA$ . Thus, if the country has a negative foreign asset position because of capital inflows ( $NFA < 0$ ), it runs a trade surplus in the steady state.

<sup>31</sup>Chien and Naknoi (2011) show that this is no longer the case if long-run growth is introduced.

<sup>32</sup>See Chien and Naknoi (2011) for more discussions on this issue when long-run growth is involved.

the second bracket are the changes in nonfinancial asset (FDI) positions. The net factor payments ( $NFP_t^h$ ) from abroad to the home country are given by

$$NFP_t^h = \left( R_{bt-1}^f \tilde{S}_t^h - \tilde{S}_t^h \right) - r_t^h u_t^f K_t^f, \quad (32)$$

where the terms inside the square bracket on the RHS are the interest rate payments from abroad and the second term on the RHS is the home country's net income payments (rents) to foreign firms for their FDI. The trade balance of the home country ( $TB_t^h$ ) can be obtained from the following accounting identity:<sup>33</sup>

$$TB_t^h = CA_t^h - NFP_t^h. \quad (33)$$

## 5 Quantitative Analysis

### 5.1 Calibration

We now calibrate the parameters in the model by taking China as the home country  $h$  and the U.S. as the foreign country  $f$ . The time period is one quarter. In our dynamic analysis, we set the initial period (before the financial liberalisation) to 1992Q1. We partition the model parameters into three sets. The first set  $\Theta_1 = \{\alpha, \beta, \delta, \psi^\ell\}$ ,  $\ell \in \{h, f\}$ , contains standard parameters and we assume they take common values across countries, with the exception of  $\psi$  which is country-specific. The second set  $\Theta_2$  contains the financial friction parameters and those pertaining to the distributions of idiosyncratic shocks. This parameter set determines the wedges in asset returns. The third set  $\Theta_3$  contains parameters pertaining to international transaction costs in capital flows. This parameter set determines the speed and scope of international capital flows given the wedges in asset returns.

To calibrate  $\Theta_1$ , we follow the standard business cycle literature (e.g. King and Rebelo, 1999) to set the discounting factor  $\beta$  to 0.985, the capital share  $\alpha$  to 0.36 and depreciation rate  $\delta$  to 0.025. We set the value of  $\psi$  such that the implied steady-state fraction of hours worked is 1/3 in each country. Since different countries have different levels of steady-state output and consumption, the implied value of  $\psi$  is thus different for China and the U.S.

The second set  $\Theta_2$  contains country-specific financial friction parameters. We calibrate their values by matching the model-implied moments in the financial autarky equilibrium to the counterparts in real data. We assume that the idiosyncratic investment efficiency shock  $\varepsilon$  and preference shock  $\theta$  follow Pareto distributions with the cumulative probability distribution (CDF)  $1 - (\varepsilon/\varepsilon_{\min}^\ell)^{-\sigma^\ell}$  and

<sup>33</sup>To be precise, aggregating the individual budget constraint in the home country gives  $(\tilde{S}_{t+1}^h - \tilde{S}_t^h) - (u_t^f K_t^f - u_{t-1}^f K_{t-1}^f) = Y_t^h - (C_t^h + \hat{I}_t^h) + (R_{bt-1}^f \tilde{S}_t^h - \tilde{S}_t^h) - r_t^h u_t^f K_t^f$  where  $\hat{I}_t^h$  is the total domestic investment including investments from both domestic firms and foreign firms and the cross-border adjustment costs. The trade balance is thus  $Y_t^h - (C_t^h + \hat{I}_t^h) = CA_t^h - NFP_t^h$ .



$1 - (\theta/\theta_{\min}^{\ell})^{-\eta^{\ell}}$  and the mean  $\mu_{\varepsilon}^{\ell} = \sigma^{\ell} / (\sigma^{\ell} - 1) \varepsilon_{\min}^{\ell}$  and  $\mu_{\theta}^{\ell} = \eta^{\ell} / (\eta^{\ell} - 1) \theta_{\min}^{\ell}$ , respectively. Note that the variance of the distribution reinforces financial frictions by affecting the precautionary saving motives of households and firms' investment rates. Given the distribution functions, the second parameter set is then given by  $\Theta_2 = \{b^{\ell}, \xi^{\ell}, \sigma^{\ell}, \mu_{\varepsilon}^{\ell}, \eta^{\ell}, \mu_{\theta}^{\ell}\}$ ,  $\ell \in \{h, f\}$ . Given that  $\mu_{\theta}$  is redundant once the other parameters are fixed, we normalise  $\mu_{\theta} = 1$  for both countries. The first two parameters pertain to borrowing limits, and we calibrate them based on household and firm finance data (if available). Specifically, for the U.S. economy,  $b^f$  captures the financial tightness on the household side. According to our model specification, in the autarky equilibrium  $b^f$  is defined as the ratio of the household borrowing to the value of equity,  $(s_{it} + Q_t) / Q_t$ . It also reflects the change in household debt relative to the change in the value of equity, i.e.,  $(\Delta \text{Household Debt}) / (\Delta \text{Household Equity})$ . According to the findings in Mian and Sufi (2011), U.S. households borrowed 25 cents on every dollar of additional home equity value in 1997, we then set  $b^f$  to 0.25. In the sensitivity analysis, we will re-calibrate  $b^f$  according to other household finance surveys, and show that the results are quite robust (see online Appendix D). The parameter in the firm's borrowing constraint  $\xi^f$  is defined as  $L_{jt} / Q_{jt}$  (or  $\Delta \text{Loan} / \Delta \text{Equity}$ ). According to the U.S. firm-level evidences in Covas and Den Haan (2011), the average ratio of the change in firm's liability to the change of firm's value of asset is 62%, so we set  $\xi^f = 0.62$ . For China, we do not have reliable information about its household debt and firm debt to pin down  $\{b^h, \xi^h\}$ . So instead we use  $\{b^h, \xi^h\}$  to target the financial development gap—the gap between the two debt-to-output ratios in China and U.S., as shown in Figure 3. Our model is not able to generate a high enough debt-to-GDP ratio for either country under any parameter values, but we can match the average gap between these ratios across the two countries shown in Figure 3.<sup>34</sup> Given that we have two parameters to match just one moment, we set one of the parameters arbitrarily, e.g., we let  $b^h$  be equal to a reasonable fraction (e.g., 1/2) of its U.S. counterpart  $b^f$ . We will show in online Appendix D that our results are not sensitive to such values.

Table 2. Targeted Aggregate Moments

	Home (China)		Foreign (U.S.)	
	Data	Model	Data	Model
Saving rate	0.40	0.40	0.18	0.18
MPK	22%	22%	11%	11%
Real interest rate	-0.71%	-0.71%	6%	6%
		Data		Model
$\frac{\text{U.S. Credit-to-GDP Ratio}}{\text{China Credit-to-GDP Ratio}}$		1.5		1.5

<sup>34</sup> As discussed in the previous sections, it is the gap between the home and foreign countries that matters for the two-way capital flows, not the absolute level of total debt-to-GDP ratio in a country.

The remaining three parameters in  $\Theta_2$ ,  $\{\sigma, \mu_\varepsilon, \eta\}$  pertain to the distributions of idiosyncratic shocks. We use three important data moments to pin them down: the aggregate saving rate, the MPK, and the risk-adjusted real interest rate in each country. We can show that a country's interest rate ( $R_b$ ), the MPK and the saving rate ( $SR$ ) under an autarky regime are jointly determined by the following three equations

$$R_b = \delta(\sigma - 1)/\sigma(\alpha/SR - 1) + 1, \quad (34)$$

$$MPK = \delta[\xi/\delta\sigma/(\sigma - 1) + \alpha/SR]^{-\frac{1}{\sigma}}/SR/\mu_\varepsilon - \delta, \quad (35)$$

$$SR = \{(b + 1)/\delta\sigma/(\sigma - 1)/[1/\Xi(R_b, \eta) - 1] + 1\}^{-1}. \quad (36)$$

where  $\Xi(R_b, \eta) = (\eta - 1)^{\frac{1}{\eta} - 1}(1/\beta/R_b - 1)^{\frac{1}{\eta}} - (1/\beta/R_b - 1)$ . These equations show that: (i) Given the  $SR$  and  $R_b$ , equation (34) pins down the parameter  $\sigma$ ; (ii) Given the MPK,  $SR$  and  $\sigma$ , equation (35) pins down the parameter  $\mu_\varepsilon$ ; (iii) Given  $SR$ ,  $R_b$  and  $\sigma$ , equation (36) pins down  $\eta$ . So we can calibrate the three parameters  $\{\sigma, \mu_\varepsilon, \eta\}$  jointly to match the three moments (i.e., average  $R_b$ , MPK and  $SR$ ) in the 1980-1991 period (long before China joining the WTO) separately for the U.S. and China (see Table 2).

Table 3. *Deep Parameter Values*

		Home (China)	Foreign (U.S.)
$\beta$	discount factor	0.985	0.985
$\alpha$	capital share in production	0.36	0.36
$\delta$	capital depreciation rate	0.025	0.025
$\xi$	borrowing constraint for firm	0.006	0.62
$b$	borrowing constraint for household	0.125	0.25
$\sigma$	shape parameter of $\varepsilon$	3.45	2.42
$\mu_\varepsilon$	mean of $\varepsilon$	0.26	0.20
$\eta$	shape parameter $\theta$	1.14	2.05
$\mu_\theta$	mean of $\theta$	1	1
$\psi$	coefficient of leisure	1.87	2.32
International Capital Flow Parameters			
A. FDI Trans. Cost: $\gamma_{kt}u_{jt}^{1+\chi}/(1+\chi)K_{jt}$			
$\bar{\gamma}_k$	steady-state value in liberalization		0.30
$\gamma_{k0}$	initial value of $\gamma_{kt}$		4.60
$\chi$	curvature		1
$\rho_k$	AR(1) coefficient of $\gamma_{kt}$ process		0.83
B. Financial Capital Trans. Cost: $\gamma_{st}s_{it}^{1+\tau}/(1+\tau)$			
$\bar{\gamma}_s$	steady-state value in liberalization		0.44
$\gamma_{s0}$	initial value of $\gamma_{st}$		0.01
$\tau$	curvature		1
$\rho_s$	AR(1) coefficient of $\gamma_{st}$ process		0.92

The third parameter set  $\Theta_3$  is related to the transaction costs of international capital flows. In the model, we specify the cross-border transaction costs for FDI and bonds as  $\gamma_k u_{jt}^{1+\chi} / (1 + \chi) K_{jt}$  and  $\gamma_s s_{it}^{1+\tau} / (1 + \tau)$ , respectively. For simplicity, we assume that the transaction costs are quadratic, i.e.,  $\chi = \tau = 1$ . The parameters  $\{\gamma_k, \gamma_s\}$  control the magnitudes of FDI flows and bond flows, therefore they represent the extent of financial liberalisation. To capture the gradualness of international market integration, we assume that  $\gamma_k$  and  $\gamma_s$  are time-varying following deterministic AR(1) processes:

$$\gamma_{kt} - \bar{\gamma}_k = \rho_k (\gamma_{kt-1} - \bar{\gamma}_k), \text{ given } \gamma_{k0}, \quad (37)$$

$$\gamma_{st} - \bar{\gamma}_s = \rho_s (\gamma_{st-1} - \bar{\gamma}_s), \text{ given } \gamma_{s0}. \quad (38)$$

The processes of  $\gamma_{kt}$  and  $\gamma_{st}$  start with some initial values and gradually achieve their long-run values  $\bar{\gamma}_k$  and  $\bar{\gamma}_s$ . So the third set of parameters are given by  $\Theta_3 \equiv \{\gamma_{k0}, \gamma_{s0}, \bar{\gamma}_k, \bar{\gamma}_s, \rho_k, \rho_s\}$ , which are calibrated to minimize the distance between model-simulated paths and actual paths of capital flows in the home country (China). In particular, the targeted series are Chinese net inward FDI position-to-GDP ( $FDI_t$ ) and total financial asset position-to-GDP ( $S_t$ ) from 1992 to 2010.<sup>35</sup> The parameters are chosen to solve the following minimisation problem

$$\hat{\Theta}_3 = \arg \min_{\Theta_3} \begin{bmatrix} FDI_t^{\text{model}} - FDI_t^{\text{Data}} \\ S_t^{\text{model}} - S_t^{\text{Data}} \end{bmatrix}' \hat{W} \begin{bmatrix} FDI_t^{\text{model}} - FDI_t^{\text{Data}} \\ S_t^{\text{model}} - S_t^{\text{Data}} \end{bmatrix}. \quad (39)$$

For simplicity, the weighting matrix  $\hat{W}$  is an identity matrix. Table 3 summarises all the calibrated parameter values.

## 5.2 Steady-State Predictions

Table 4 reports the predictions of the model under the financial autarky and liberalisation regimes. Columns 1 and 2 pertain to China and the ROW (represented by the U.S.) in the autarky regime. Columns 3 to 8 pertain to the two countries in the financial liberalisation regime (with partial and full liberalisation, respectively).

In the autarky regime, China has a higher Tobin's  $q$  than the U.S. (4.99 vs. 1.79), a higher annual rate of return to fixed capital (22% vs. 11%), but a lower interest rate (-0.71% vs. 6.0%). All these gaps are predicted by our model because of the gaps in financial development between China and the U.S.<sup>36</sup>

<sup>35</sup>The data series are calculated from the updated dataset of Lane and Milesi-Ferretti (2007).

<sup>36</sup>Due to the modelling strategy we choose to simplify computations, the notion of debt in our model corresponds only to short-term debts, including the one-period government bond on the household side and the intra-period debt on the firm side. Hence, it makes sense to compare our model's prediction only with the short-term debt in the data. In our benchmark calibration, the model's prediction for the debt-to-GDP ratio in the U.S. economy is 18%, while for

Because of the cross-country spread in the rates of return to financial and fixed capital, financial liberalisation between countries will induce China to hold negative foreign productive asset positions (FDI inflow) but positive financial asset positions (bond outflow), with the former equal to -19.85% of GDP and the latter equal to 53.06% of GDP in the steady state. The model-implied net FDI position is close to the actual value, while the financial asset position is about 15% higher than that in the data,<sup>37</sup> indicating that the Chinese economy may not actually be in its steady state yet in the financial liberalisation regime.

Table 4. *Steady States in Financial Autarky and Financial Liberalization*

	Autarky		Only Financial Capital Flows		Only Fixed Capital Flows		Both Capital Flows	
	(1) China	(2) ROW	(3) China	(4) ROW	(5) China	(6) ROW	(7) China	(8) ROW
Rate of return to fixed K (MPK) (%)	22.00	11.00	22.62	10.94	20.22	11.05	20.64	11.01
Real interest rate (%)	-0.71	6.00	-0.55	5.96	-1.15	6.03	-1.05	5.99
Tobin's $q$	4.99	1.79	4.96	1.79	5.07	1.79	5.05	1.79
Net foreign asset positions (%GDP)	...	...	52.33	-38.26	-17.39	14.78	33.21	-26.28
Direct investment abroad (%GDP)	...	...	...	...	-17.39	14.78	-19.85	15.71
Bonds (%GDP)	...	...	52.33	-38.26	...	...	53.06	-41.99
Trade imbalances (%GDP)	...	...	-3.05	2.23	5.25	-4.47	2.97	-2.35
Interest rate payments (%GDP)	...	...	-3.05	2.23	...	...	3.12	-2.47

To better understand the different impacts of financial and fixed capital flows on each country's MPK and interest rate, we consider first the partial liberalisation scenario by allowing only one type of capital (bonds or fixed capital) to move across borders. Columns 3 and 4 report the steady state in which only financial assets (bonds) are internationally mobile. The opening up of bond markets induces financial capital outflow from China to the ROW, which makes the interest rates converge somewhat across countries—the convergence is not high because of transaction costs in international capital flows (or home bias). As a result, the equilibrium real interest rate in China increases from -0.71% in autarky to -0.55%, which further raises the domestic rate of return to fixed capital from 22% to 22.62%. The situation in the ROW is the opposite: the real interest rate decreases slightly from 6.0% in autarky to 5.96%, and the MPK declines from 11% in autarky

China it is 12%. Based on the U.S. flow of funds data, the short-term debt-to-GDP ratio is around 27%. For Chinese economy, the People's Bank of China reports the short-term loans each year. However, in China the major part of debt goes to state-owned enterprises (SOEs) because of the lack of sufficiently developed private banking system and private bond market for firms to issue debt (consistent with our model); thus most credit loans are channeled by the state-owned banking system and the bulk of such loans goes to the SOEs. In 2013 the total debt-to-GDP ratio for SOEs was around 104%, while the domestic credit-to-GDP ratio (according to WDI) was around 140%. That is, the debt borrowed by SOEs accounts at least 70% of the overall total debt in China. If we assume that the fraction of SOEs' short-term debt in total short-term debt is the same as the that of the total debt, then the short-term debt to non-SOEs is around 15% of China's GDP, closely matching our model's predictions (12%). So our model can explain about 80% of China's short-term debt and 70% of US short-term debt.

The most important thing to notice, however, is that it is the relative gap of financial development (debt-to-GDP ratio) between China and the US that matters for our results, not the absolute level of debt-to-GDP ratio in each country when it comes to international capital flows. This is why in the calibration we do not particularly target the absolute country-specific debt-to-GDP ratio, but only the gap between the two countries' debt-to-GDP ratios.

<sup>37</sup>By 2010, the accumulative net inward FDI of China was about 21% of GDP, and the net outward debt position was about 46% of GDP.

to 10.94%. These results confirm our previous discussions that international financial asset flows narrow the gap in country-specific interest rates but enlarge the gap in the rate of return to fixed capital.

Columns 5 and 6 report the situation when only fixed capital can move internationally. Fixed capital will flow from the ROW to China because China has a higher MPK in autarky. In particular, FDI inflows increase the capital supply in China, thereby reducing China's domestic interest rate from -0.71% to -1.15%. The inward FDI flow also reduces the equilibrium MPK in China from 22% to 20.22%. In contrast, FDI outflows raise the interest rate in the ROW from 6.0% in autarky to 6.03%, and pushes up ROW's MPK from 11% to 11.05%.

The outcome with both types of capital flows are reported in columns 7 and 8. Note that neither MPK nor the interest rate changes significantly from their autarky values. These predictions are consistent with the data in Figure 2 where the gaps in MPKs and risk-adjusted real interest rates remain roughly constant over time (e.g., both before and after China joined the WTO in 2001). Our model is consistent with this fact despite the fact that financial liberalisation in our model can trigger enormous amount of capital flows in the steady state. The reasons may be as follows. On the data part, China has capital controls and an essentially fixed (or managed floating) exchange rate with the U.S. dollar. Hence, the interest rates in the two countries are never equalised. Thus, small wonder that they still show significant differences even today.<sup>38</sup> This persistent interest rate gap as well as the persistent gap in MPK are captured in our model by the international transaction costs. Therefore, despite the large two-way capital flows across countries, the gaps between the two countries' asset returns are not significantly reduced. In other words, our quantitative analysis shows that China's saving glut does not significantly reduce the real interest rate in the U.S. when our model is calibrated to match the actual amount of financial capital outflows from China to the U.S.

Financial capital outflow tends to increase the domestic interest rate and fixed capital return, whereas FDI inflow has the opposite effect. Since international capital flows are fully determined by the cross-country discrepancies in interest rate and MPK, the two types of capital flows reinforce each other—FDI inflows may cause financial capital outflows, and financial capital outflows in turn may cause FDI inflows. The middle panel in Table 4 labeled "Net foreign asset positions" quantitatively illustrates this point. When only bonds can be traded across countries, the financial capital outflow from China is 52.33% of its GDP. When only fixed capital is allowed to move across countries, FDI inflows to China is 17.39% of its GDP. The corresponding ratios increase significantly when the two capital markets are both liberalised. The financial capital outflows from China become 53.06% of its GDP, rising 0.73 percentage points in GDP share compared to the

<sup>38</sup>Figure 2B shows a temporary convergence between the two interest rates in 2004 but they diverged again after that.

value in partial liberalisation. Meanwhile, the FDI inflows rise to 19.85% of China's GDP, which is more than 2.4 percentage points higher in its GDP share.

The bottom panel in Table 4 reports the impact of capital flows on trade balances. When only financial assets are mobile across countries, China will run a trade deficit (-3.05% of GDP) in the long run. In contrast, the ROW (e.g., the U.S.) will run a trade surplus (2.23% of GDP). These trade imbalances come entirely from interest payments on international bonds. However, when only fixed capital is mobile across countries, China will run a trade surplus (5.25% of GDP) in the steady state while ROW will run trade deficits (-4.47% of GDP). These balances come entirely from capital gains from FDI positions (i.e., net FDI outflows from ROW to China). Because the rate of return to FDI dominates the rate of return to financial assets, in the full liberalisation regime China will maintain a long-run trade surplus of 2.97% of its GDP while ROW (e.g., the U.S.) will maintain a trade deficit of -2.35% of its GDP. Therefore, even though FDI flows are smaller than financial asset flows in GDP shares (both in the data and in the model), developed countries can have permanent trade deficits with developing countries because FDI payments from developing countries are much larger than interest payments on bonds from developed countries.

### 5.3 *Transitional Dynamics*

Figure 6 shows the transitional dynamics of major aggregate variables when the model economy opens up from financial autarky to financial liberalisation (with both financial and fixed capital flows). The figure shows a typical pattern of diverging trends in two-way capital flows: financial assets leave China and flow into the ROW (grey line in the second panel on the left column), while FDI leaves the ROW and flows into China (grey line in the top panel on the right column). Because the volume of financial asset flows dominates that of FDI flows, the net foreign asset position is positive in China and negative in the ROW after about nine years (top left panel), explaining the reverse capital flow pattern discussed in the introduction.

Despite positive net capital outflows (financial plus fixed), China runs a trade surplus that increases over time (grey line in the third panel in the right column), reaching 7.5% of GDP ten years after liberalisation. The trade surplus gradually declines to 3% in the steady state. However, the current account in China decreases briefly, then increases in the following years and eventually balances itself in the long run (grey line in the second panel on the right column). Since FDI earns a much higher rate of return than bonds, China always receives negative net income payments. As a result, China runs a permanent trade surplus while the ROW runs a permanent trade deficit. Hence, our model suggests that the global imbalances in world trade are sustainable in the long run even if the current accounts are perfectly balanced. Obstfeld and Rogoff (2007) have argued that a permanent trade imbalance is unsustainable, thus predicting that a reversal of the U.S. current

account deficit is inevitable and that the future U.S. trade surplus requires substantial depreciation of the dollar's real exchange rate. Our model predicts instead that the U.S. is able to sustain a trade deficit of about 2% of GDP permanently with China unless financial markets in China develop to the same degree as those in the U.S.

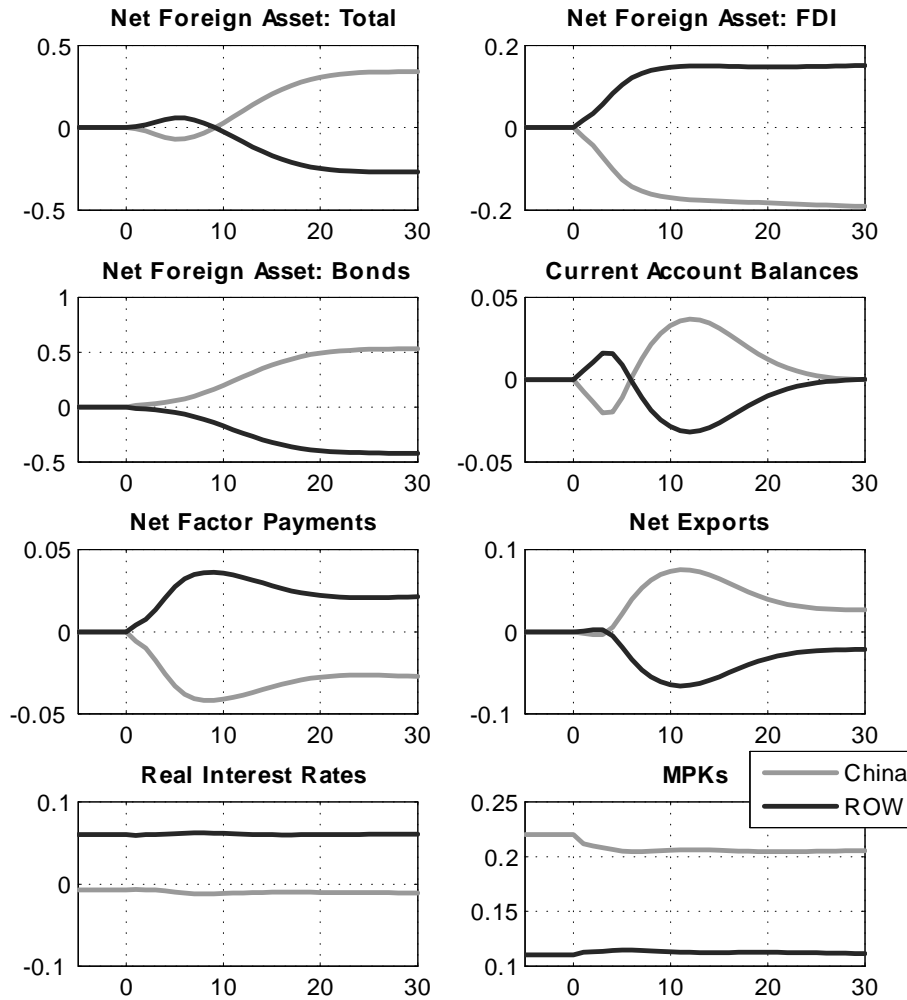


Figure 6. *Transitional Dynamics after Financial Liberalisation*

The smoothly increasing trends in the transitional dynamics of the two-way capital flows and current accounts are due partially to consumption smoothing and partially to the gradual liberalisation process assumed in the transaction costs. But even if we eliminate the transaction costs and allow rapid liberalisation, we would still generate smooth rising trends in financial capital flows and current accounts (see online Appendix D.2). The reason for this is that a large and dramatic change in financial capital flows would mean a large and dramatic change in consumption, so the households have incentives to smooth consumption by smoothing international borrowing.

In addition, the bottom two panels show that the interest and the MPK are not largely affected under two-way capital flows, despite the colossal volume of capital flows after liberalisation. There

are two major reasons. First, financial liberalisation per se does not eliminate the international transaction costs in capital flows or reduce the domestic borrowing limits in either economy. On the top of this, two-way capital flows reinforce these gaps because FDI inflows and financial asset outflows have the opposite effects on the interest rate and the MPK, offsetting their impacts on these asset returns. These two factors together generate the sustained gaps in asset returns between the two countries. It is, however, possible to find parameter configurations such that the U.S. interest rate can be significantly reduced by financial capital inflows from China (provided that the international transaction cost of bonds is small and the autarky interest rate in the U.S. is far from  $1/\beta$  due to severe financial frictions on the household side). But such parameter configurations would then prevent our model from quantitatively matching the volume and dynamic paths of two-way capital flows observed in the data. This brings us to Figure 7.

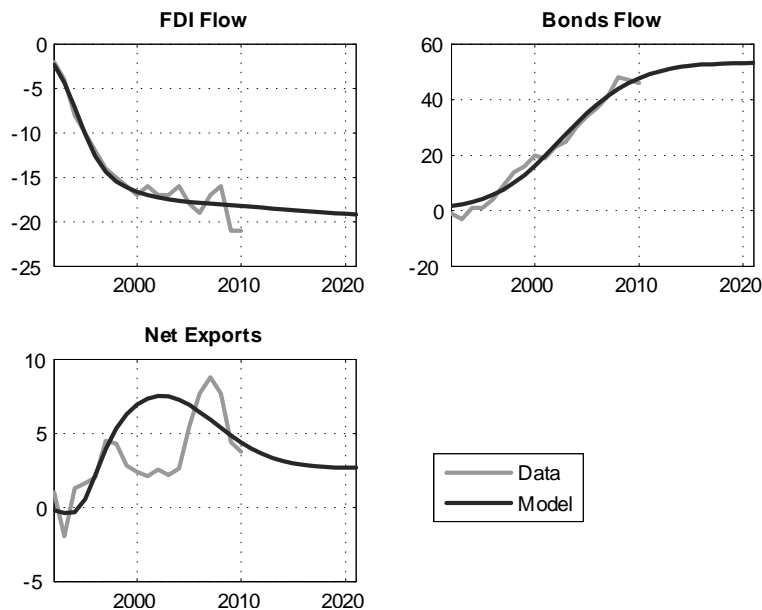


Figure 7. *Net Foreign Asset Positions in China: Model vs. Data*

Figure 7 compares the model-simulated dynamic paths of capital flows in the home country (dark lines) and their actual paths in China (grey lines), where the vertical axes represent the percentage of GDP. The top-left panel shows net FDI inflows into China, the top-right panel shows net financial asset outflows from China, and the bottom panel shows net exports of China. The simulated series closely track the trends in Chinese data, indicating that the model can explain the dynamics of China's capital flows very well. Although our calibrations do not target net exports, the simulated path captures the general trend in the data. Moreover, the simulated series predict that by 2020 China's accumulated FDI inflows and foreign reserves (financial asset outflows) will reach about 20% and 53% of GDP, respectively. This will take another 10 years to accomplish (our sample period ends at 2010). The model also predicts that China's net exports will level off and



gradually reach about 3% of GDP after 2017. Given the recent trends in the Chinese trade data, this prediction seems credible.

## 6 Conclusions

Capital flows both ways instead of one way between the North and the South: fixed capital flows from rich to poor countries whereas financial capital flows in the opposite direction. We augment the standard neoclassical growth model with two wedges (a saving wedge and an investment wedge) to quantitatively explain the magnitude of the two-way capital flows. We show that severe financial frictions in poor countries—the lack of an efficient banking-credit system in particular—can lead to insufficient investment on the firm side (the investment wedge) and excessive saving on the household side (the saving wedge). Consequently, fixed capital is scarcer while financial capital is relatively abundant in the South, creating a gap between the MPK and the real interest rate both within and across countries. This gap in assets returns drives the observed two-way capital flows between the North and the South.

Our contributions are therefore fourfold: (i) the ability to make a clear distinction between financial capital flows and fixed capital flows in a full-fledged two-country neoclassical growth model with double-heterogeneous agents; (ii) the ability to disentangle the interest rate from the MPK through Tobin's  $q$  theory and show that the market rate of return to fixed capital can be over 20% a year in equilibrium despite low interest rates (as in China);<sup>39</sup> (iii) the ability to explain China's excessively high aggregate saving rate of 40% (despite low interest rates) and its massive trade imbalances with the ROW; and (iv) the ability to provide a tractable tool for evaluating the welfare consequences of the two fundamentally different forms of capital flows.<sup>40</sup>

Our main findings challenge the conventional wisdom in the global imbalance literature in several ways. For example, our model predicts that permanent global trade imbalances are sustainable (with the North running deficits and the South running surpluses). Also, our quantitative analysis shows that the impact of massive financial capital flows (from emerging markets to developed economies) on the world interest rate can be quantitatively small and negligible, in sharp contrast to the conjecture of Bernanke (2005). Another implication of our analysis is that the reduction of global imbalances (for better or worse) hinges neither on adjusting the exchange rates nor on capital account liberalisation, but rather on improving emerging economies' banking system (i.e., reducing borrowing constraints facing both households and firms) so that household savings in the

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<sup>39</sup>In contrast to the existing approaches of studying FDI (e.g., MQR 2009), where FDI is modeled as households' portfolio choices through risky equity investment, we model FDI as firms' production decisions through international factor allocation. Therefore, instead of creating the differential rates of return between bonds and FDI through equity premium, we achieve this through Tobin's  $q$  theory—a standard approach in line with the neoclassical investment theory.

<sup>40</sup>Interested readers should refer to our working paper (Wang, Wen and Xu, 2013).

South can be channeled more effectively to its domestic production sector.

However, our model does not fully resolve the "allocation puzzle" of Gourinchas and Jeanne (2013) because we have not shown why countries with faster growth tend to attract less international capital. To fully resolve the "allocation puzzle", we need to introduce growth into our two-country model and show that the cross-country gaps in the financial interest rate and the MPK are increasing (rather than decreasing) functions of the growth rate (at least in the short run). This is beyond the scope of this paper and is thus left for future work (see Wen, 2009, 2011, for critical progress in this direction).

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# Appendix

## A Decision Rules and System of Equations

### A.1 A Single Firm's Decision Rules

We define MPK (net of depreciation rate  $\delta$ ) as  $MPK_t = R_{kt} - \delta$ . The following proposition shows that  $\{r_t, R_{kt}\}$  are both independent of firms' idiosyncratic shocks and are closely related to each other.

**Proposition 3** Given  $\{r_t, r_t^f\}$ , the optimal FDI decision ( $u_{jt}$ ) is given by

$$u_{jt} = \begin{cases} 0 & \text{if } r_t^f \leq r_t \\ \left[ (r_t^f - r_t) / \gamma_k \right]^{\frac{1}{\chi}} & \text{if } r_t^f > r_t \end{cases}, \quad (\text{A.1})$$

and the MPK is determined by

$$MPK_t \equiv R_{kt} - \delta = r_t + \mathbf{1}_{r_t^f > r_t} \left[ \chi / (1 + \chi) \gamma_k^{-\frac{1}{\chi}} (r_t^f - r_t)^{\frac{1+\chi}{\chi}} \right] - \delta, \quad (\text{A.2})$$

where  $\mathbf{1}_{r_t^f > r_t}$  is an index function that takes a value of 1 whenever  $r_t^f > r_t$  and a value of 0 otherwise.

**Proof.** See online Appendix B.1. ■

A firm's FDI decision depends completely on the spread of MPK between the two countries. It can be shown easily that the function  $MPK$  is strictly increasing in  $r_t$  and weakly increasing (non-decreasing) in  $r_t^f$ . Because of the constant returns to scale (CRS) production function and i.i.d. investment-efficiency shocks, both  $R_{kt}$  and  $r_t$  are independent of firms' idiosyncratic shocks. Based on this important property, we conjecture that the value of a firm is given by the following functional form suggested by Hayashi (1982):

$$V_t(K_{jt}) = v_t K_{jt}, \quad (\text{A.3})$$

where  $v_t$  is the average (and marginal) value of a firm and depends only on the aggregate states. Hence, it is free of the firm index  $j$ . We define  $q_t = \rho E_t (\Lambda_{t+1} / \Lambda_t) v_{t+1}$ , which is the conventional measure of Tobin's  $q$ . With the conjectured value function, the firm's investment problem becomes

$$v_t K_{jt} = \int \max_{I_{jt}} \{ R_{kt} K_{jt} - I_{jt} + q_t [(1 - \delta) K_{jt} + \varepsilon_{jt} I_{jt}] \} d\Phi, \quad (\text{A.4})$$

subject to the constraints (20), (22), and

$$L_{jt} \leq q_t \xi K_{jt}. \quad (\text{A.5})$$

**Proposition 4** *There exists a cutoff  $\bar{\varepsilon}_t = \frac{1}{q_t}$ , such that the firm's optimal investment decisions follow a trigger strategy:*

$$I_{jt} = \begin{cases} q_t \xi K_{jt} + R_{kt} K_{jt} & \text{if } \varepsilon_{jt} > \bar{\varepsilon}_t \\ 0 & \text{otherwise} \end{cases}. \quad (\text{A.6})$$

*In addition, the marginal value of the firm is given by*

$$v_t = R_{kt} + (1 - \delta) q_t + (q_t \xi + R_{kt}) \Omega(q_t), \quad (\text{A.7})$$

*where  $\Omega(q_t) \equiv \int_{\varepsilon_{jt} > 1/q_t} (q_t \varepsilon_{jt} - 1) d\Phi$  with  $\Omega'(q_t) > 0$ , and Tobin's  $q(q_t)$  evolves according to*

$$q_t = \rho(\Lambda_{t+1}/\Lambda_t) [R_{kt+1} + (1 - \delta)q_{t+1} + (q_{t+1}\xi + R_{kt+1})\Omega(q_{t+1})]. \quad (\text{A.8})$$

**Proof.** See online Appendix B.2. ■

Briefly speaking,  $v_t$  is the value of one unit of existing capital and  $q_t$  is the value of one unit of newly installed capital. The marginal benefit of new investment is thus  $q_t \varepsilon_{jt}$ . Since the real cost of investment is 1, investment is profitable if and only if  $q_t \varepsilon_{jt} > 1$  or  $\varepsilon_{jt} > \bar{\varepsilon}_t \equiv 1/q_t$ , which defines the cutoff. In such a case, the firm is willing to borrow as much as possible to invest, so its borrowing constraint binds. This explains the investment decision rule in equation (A.6).

By definition,  $q_t$  equals the discounted future value of one unit of capital in the next period  $\rho(\Lambda_{t+1}/\Lambda_t) v_{t+1}$ , which is equation (A.8) after substitution using equation (A.7). The average (marginal) value of the firm ( $v_t$ ) consists of three parts on the right-hand side of equation (A.7). First, one unit of capital can generate  $R_{kt}$  units of operating profit in period  $t$ . Second, one unit of capital can carry  $1 - \delta$  remaining units to the next period with value  $(1 - \delta) q_t$  after depreciation. Finally, the capital can also be used as collateral. With probability  $1 - \Phi(1/q_t)$ , the firm has a profitable investment opportunity and one unit of capital is able to obtain  $q_t \xi$  units of loans, which can expand investment by  $(q_t \xi + R_{kt})$  units by equation (A.6). After repaying the loans at zero interest rate, the net value of the loan is  $(q_t \varepsilon_{jt} - 1)$ ; hence, the value of the collateral is  $(q_t \xi + R_{kt}) \int_{\varepsilon_{jt} > 1/q_t} (q_t \varepsilon_{jt} - 1) d\Phi$ . This explains equation (A.8).

## A.2 A Single Household's Decision Rules

**Proposition 5** *The optimal demand for foreign bond holdings  $\tilde{s}_{it+1}$  is given by*

$$\tilde{s}_{it+1} = \begin{cases} 0 & \text{if } R_{bt} \geq R_{bt}^f \\ \left[ (R_{bt}^f - R_{bt}) / \gamma_s \right]^{\frac{1}{\tau}} & \text{if } R_{bt} < R_{bt}^f \end{cases}. \quad (\text{A.9})$$

Further, arbitrage among financial assets implies that the portfolio's price satisfies

$$Q_t = (Q_{t+1} + D_{t+1}) / R_{bt}. \quad (\text{A.10})$$

Namely, the risk-free rate is the proper discounting factor for the firms.

**Proof.** See online Appendix B.3. ■

The demand for foreign bonds is an increasing function of the cross-country interest spread,  $R_{bt}^f - R_{bt}$ , provided that the spread is positive. The parameter  $\gamma_s$  determines the cost of holding foreign bonds; it thus represents the extent of capital controls or transaction costs in the international bonds market. Financial autarky for bond trading is obtained if  $\gamma_s = \infty$ . In the limit as  $\gamma_s \rightarrow 0$ , the two interest rates,  $R_{bt}^f$  and  $R_{bt}$ , must be equalised in general equilibrium, so the model reduces to the standard setting with a single international bond.

Note that even households with large preference shocks (or with a strong urge to consume) may still hold positive amount of foreign bonds  $\tilde{s}_{it+1}$  provided that  $R_{bt}^f > R_{bt}$ , because they can borrow from the domestic bond market, i.e.,  $s_{it+1} < 0$ . More importantly, equation (A.9) implies that the country with a lower interest rate will have positive net outflows in financial capital. Thus, to show the direction of financial capital flows, we only need to compare the interest rates in the two countries.

Denoting by

$$H_{it} = (Q_t + D_t) a_{it} + W_t n_{it} + R_{bt-1} s_{it} + R_{bt-1}^f \tilde{s}_{it} - \gamma_s (\tilde{s}_{it})^{1+\tau} / (1 + \tau) \quad (\text{A.11})$$

the gross wealth of household  $i$  in period  $t$ , the following proposition shows that a household's consumption-saving decisions follow simple rules and that the distribution of gross wealth is degenerate across households (i.e.,  $H_{it} = H_t$  for all  $i$ ).

**Proposition 6** *Given the real wage  $W_t$  and the real interest rate  $R_{bt}$ , the optimal consumption and saving of household  $i$  are given, respectively, by*

$$c_{it} = \min \{ \theta_{it} / \bar{\theta}_t, 1 \} (H_t + B_t), \quad (\text{A.12})$$

$$s_{it+1} + \tilde{s}_{it+1} + a_{it+1} Q_t = \max \{ (\bar{\theta}_t - \theta_{it}) / \bar{\theta}_t, 0 \} (H_t + B_t) - B_t, \quad (\text{A.13})$$

where the target wealth  $H_t$  and the cutoff  $\bar{\theta}_t$  are identical across households and jointly determined by the following two equations:

$$\bar{\theta}_t = \beta R_{bt} (H_t + B_t) E_t (\psi / W_{t+1}), \quad (\text{A.14})$$

$$W_t \int \max(\theta, \bar{\theta}_t) / (H_t + B_t) d\Phi = \psi. \quad (\text{A.15})$$

**Proof.** See online Appendix B.4. ■



### A.3 *Wedges and System of Aggregate Dynamic Equations*

As we have illustrated already with our toy model, financial frictions introduce two wedges into our model compared with standard representative-agent neoclassical growth models. The saving wedge is introduced by borrowing constraints on the household side, and the investment wedge is introduced by borrowing constraints on the firm side. These wedges lead to low returns to household savings (financial interest rate) and high returns to firm investment (MPK), thus creating the driving forces of international two-way capital flows. To derive these wedges explicitly, consider the effective interest rate facing an "aggregate" (average) household and the effective rate of return to capital facing an "aggregate" (average) firm.

The CRS production technology implies that the equilibrium factor prices are  $W_t^\ell = (1 - \alpha) Y_t^\ell / n_t^\ell$  and  $r_t^\ell = \alpha Y_t^\ell / \bar{K}_t^\ell$ , where the aggregate output  $Y_t^\ell = (\bar{K}_t^\ell)^\alpha (n_t^\ell)^{1-\alpha}$  and the aggregate capital stock  $\bar{K}_t^\ell = u^{\ell c} K_t^{\ell c} + (1 - u_t^\ell) K_t^\ell$ . After aggregating households' decisions in equations (A.12) and (A.13) as well as the budget constraint,<sup>41</sup> and combining with equations (A.14) and (A.15), we obtain

$$\Psi(\bar{\theta}_t^\ell) \bar{\theta}_t^\ell / C_t^\ell = \beta R_{bt}^\ell \left[ \Psi(\bar{\theta}_{t+1}^\ell) \bar{\theta}_{t+1}^\ell / C_{t+1}^\ell \right] G(\bar{\theta}_{t+1}^\ell), \quad (\text{A.16})$$

$$\left[ \Psi(\bar{\theta}_t^\ell) \bar{\theta}_t^\ell / C_t^\ell \right] W_t^\ell G(\bar{\theta}_t^\ell) = \psi^\ell, \quad (\text{A.17})$$

where

$$G(\bar{\theta}_t^\ell) = \int \max(\theta / \bar{\theta}_t^\ell, 1) dF(\theta) > 1 \quad (\text{A.18})$$

captures the liquidity premium of cash flows and  $\Psi(\bar{\theta}_t^\ell) = \int \min(\theta / \bar{\theta}_t^\ell, 1) dF(\theta)$  captures the marginal propensity to consume. Equation (A.16) corresponds to the intertemporal Euler equations for consumption and saving and equation (A.17) to aggregate labor supply. If  $\Psi(\bar{\theta}_t^\ell) \bar{\theta}_t^\ell / C_t^\ell$  is treated as the aggregate marginal utility of consumption, then the savings wedge introduced by the financial friction on the household side is captured by the function  $G(\bar{\theta})$ . Because  $G(\bar{\theta}) > 1$ , the equation shows that the interest rate is lower than the rate of time preference ( $\beta R_b < 1$ ), suggesting that financial friction induces higher saving (Aiyagari, 1994). The labor supply equation shows that financial friction induces a higher labor supply. The intuition is that the positive probability of a binding borrowing constraint induces the agent to work harder to provide enough liquidity to reduce that probability. This means that the effective rate of return to labor is the real wage compounded by the liquidity premium  $G(\bar{\theta})$ .

<sup>41</sup>The individual budget constraint is  $c_{it} + s_{it+1} + \tilde{s}_{it+1} + a_{it+1} Q_t = H_{it}$ , where  $H_{it}$  is defined in equation (A.11).

On the firm side, the Euler equation for capital investment is

$$q_t^\ell = \left[ R_{kt+1}^\ell + (1 - \delta)q_{t+1}^\ell + (q_{t+1}^\ell \xi^\ell + R_{kt+1}^\ell) \Omega(q_{t+1}^\ell) \right] / R_{bt}^\ell. \quad (\text{A.19})$$

Notice that if  $\Omega(q_t^\ell) = 0$  and  $q_t = 1$ , the above equation is simply a modified neoclassical first-order condition with respect to capital investment. Therefore  $\Omega(q_t^h) > 0$  together with  $q_t > 1$  captures the investment wedge. It can be shown that Tobin's  $q(q_t)$  measures the gap between the MPK and the financial interest rate.

The equilibrium dynamics of the model are characterised by a system of dynamic rational expectations equations in terms of aggregate variables. Besides the above wedge equations representing financial frictions, the rest of the aggregate equations pertaining to the aggregate resource constraint, aggregate production function, aggregate capital accumulation, aggregate consumption, and aggregate investment are given, respectively, by

$$C_t^\ell + S_{t+1}^\ell + \tilde{S}_{t+1}^\ell + I_t^\ell + r_t^\ell u_t^{\ell c} K_t^{\ell c} + \gamma_s (\tilde{S}_{t-1}^\ell)^{1+\tau} / (1 + \tau) + \gamma_k (u_t^\ell)^{1+\chi} / (1 + \chi) K_t^h = Y_t^\ell + R_{bt-1}^\ell S_t^\ell + R_{bt-1}^{\ell c} \tilde{S}_t^\ell \quad (\text{A.20})$$

$$Y_t^\ell = \left[ u^{\ell c} K_t^{\ell c} + (1 - u_t^\ell) K_t^\ell \right]^\alpha (n_t^\ell)^{1-\alpha}. \quad (\text{A.21})$$

$$K_{t+1}^\ell = \Gamma(q_t^\ell) / \pi(q_t^\ell) I_t^\ell + (1 - \delta) K_t^\ell, \quad (\text{A.22})$$

$$C_t^\ell = 1 / \left[ 1 / \Psi(\bar{\theta}_t^\ell) - 1 \right] (q_t^\ell K_{t+1}^\ell + S_{t+1}^\ell + \tilde{S}_{t+1}^\ell + B_t^\ell), \quad (\text{A.23})$$

$$I_t^\ell = \pi(q_t^\ell) \left( R_{kt}^\ell + q_t^\ell \xi^\ell \right) K_t^\ell, \quad (\text{A.24})$$

where  $\pi(q_t^\ell) \equiv 1 - \Phi(1/q_t^\ell)$  and  $\Gamma(q_t^\ell) \equiv \int_{\varepsilon > 1/q_t^\ell} \varepsilon d\Phi(\varepsilon)$ . The cutoffs  $\{\bar{\theta}_t^\ell, 1/q_t^\ell\}$  provide sufficient statistic for the distribution of households and firms' allocations. To facilitate analysis, we assume that the borrowing limit of the households is proportional to some aggregate variables,  $B_t^\ell = b^\ell q_t^\ell K_t^\ell$ , where the parameter  $b^\ell \geq 0$  measures the tightness of borrowing constraints on the household side. A specific borrowing limit such as this permits balanced growth and facilitates steady-state calibrations. The total household income in equation (A.20) comes from several sources: total domestic output, returns from domestic bonds, and returns from foreign bonds. The aggregate consumption in equation (A.23) is proportional to total saving and borrowing limits ( $B_t$ ). The aggregate investment is obtained through aggregating equation (A.6). For the financial autarky regime, we also add

$$S_{t+1}^\ell = \tilde{S}_{t+1}^\ell = u_t^\ell = 0. \quad (\text{A.25})$$

For the financial liberalisation regime, we also add

$$S_{t+1}^\ell + \tilde{S}_{t+1}^{\ell c} = 0, \quad (\text{A.26})$$

$$\tilde{S}_{t+1}^\ell = \mathbf{1}_{R_{bt}^{\ell c} > R_{bt}^\ell} \left[ \left( R_{bt}^{\ell c} - R_{bt}^\ell \right) / \gamma_s \right]^{\frac{1}{\tau}}, \quad (\text{A.27})$$

$$u_t^\ell = \mathbf{1}_{r_t^{\ell c} > r_t^\ell} \left[ \left( r_t^{\ell c} - r_t^\ell \right) / \gamma_k \right]^{\frac{1}{\chi}}. \quad (\text{A.28})$$

The system of equations (A.16)-(A.28) consists of 22 equations that determine the dynamic equilibrium path of 22 endogenous aggregate variables,  $\{K_t^\ell, n_t^\ell, I_t^\ell, Y_t^\ell, q_t^\ell, C_t^\ell, \bar{\theta}_t^\ell, S_t^\ell, \tilde{S}_t^\ell, R_{bt}^\ell, u_t^\ell\}$ , for  $\ell = \{h, f\}$ . The transitional equilibrium path from autarky to financial liberalisation can all be computed in a straightforward manner by standard numerical methods in the representative-agent model literature.

#### A.4 Aggregate Demand and Supply of Capital

Deriving the aggregate demand and supply functions of capital can help us understand the subtle issues regarding the conditions of two-way capital flows in Section 4. We first derive the steady-state demand function for aggregate capital in the home country based on firms' investment behaviors. From the evolution equations of Tobin's  $q$  (A.19) and capital stock (A.22) as well as the aggregate investment (A.24) in Appendix A.3, we obtain the following two equations that implicitly describe the gross rate of return to fixed capital  $R_k$  (MPK) and Tobin's  $q$  as functions of the real interest rate  $R_b$ :

$$\delta = (R_k + q\xi) \Gamma(q), \quad (\text{A.29})$$

$$R_k = q(R_b - 1) + \delta\pi(q) / \Gamma(q). \quad (\text{A.30})$$

where  $\pi(q) = 1 - \Phi(1/q)$ , and  $\Gamma(q) = \int_{\varepsilon > 1/q} \varepsilon d\Phi$ . The term  $\Gamma(q) / \pi(q)$  on the RHS of the last equation is the average investment efficiency for active firms; thus, it is increasing in the cutoff  $1/q$  or decreasing in  $q$ . This equation suggests that the Tobin's  $q$  ( $q$ ) measures the spread between the return to fixed capital and the return to financial capital (the interest rate). Indeed, if we assume that the efficiency shock  $\varepsilon_j$  follows a binomial distribution with only two realisations, 0 and 1, then the above equation reduces to  $R_k - \delta = q(R_b - 1)$ . In this case,  $q$  is exactly the wedge between the rate of return to fixed capital (MPK) and the real interest rate.

Combining equations (A.29) and (A.30), we can solve  $R_k$  and  $q$  as functions of the interest rate  $R_b$  and the financial development parameter  $\xi$ :  $R_k \equiv \mathbb{R}(R_b, \xi)$ ,  $q \equiv \mathbb{Q}(R_b, \xi)$ . The following

proposition shows that given the interest rate, the country with lower financial development ( $\xi$ ) on the firm side tends to have both a higher Tobin's  $q$  and a higher MPK.

**Proposition 7** *The function of MPK  $\mathbb{R}$  is strictly increasing in the interest rate  $R_b$  and strictly decreasing in the financial development  $\xi$ , that is,  $\partial \mathbb{R}(R_b, \xi) / \partial R_b > 0$  and  $\partial \mathbb{R}(R_b, \xi) / \partial \xi < 0$ . The function of Tobin's  $q$  ( $\mathbb{Q}$ ) is strictly decreasing in both the interest rate  $R_b$  and the financial development  $\xi$ , that is,  $\partial \mathbb{Q}(R_b, \xi) / \partial R_b < 0$  and  $\partial \mathbb{Q}(R_b, \xi) / \partial \xi < 0$ .*

**Proof.** Based on the fact that both  $\Gamma(q)$  and  $\pi(q)/\Gamma(q)$  are strictly increasing in  $q$ , the results can be obtained easily through the implicit function theorem. ■

Our model predicts that the MPK and financial interest rate are positively correlated but the correlation is not perfect—there is a wedge between the two and the magnitude of this wedge (Tobin's  $q$ ) depends crucially on the degree of financial development. The wedge is smaller and the correlation is stronger for countries that are financially more developed. These predictions are consistent with the empirical findings of Ohanian and Wright (2007).

To obtain the aggregate capital demand function, we need to link the MPK to the capital-to-output ratio. According to the previous discussions, the LHS of equation (A.30) is increasing in  $r_t$  and thus decreasing in the capital-to-output ratio  $\frac{\bar{K}}{\bar{Y}}$ . On the other hand, Proposition 7 implies that the RHS of equation (A.30) is increasing in the interest rate  $R_b$ . Therefore, equation (A.30) implicitly describes the aggregate capital demand (or the capital-to-output ratio) as a downward sloping function of the interest rate.

We now derive the aggregate capital supply from the household. From equation (A.17) in Appendix A.3, the cut-off  $\bar{\theta}$  is implicitly determined by

$$\beta R_b G(\bar{\theta}) = 1. \quad (\text{A.31})$$

Since  $G$  is a decreasing function of  $\bar{\theta}$ , equation (A.31) implies the cutoff is increasing in  $R_b$ . Since  $G > 1$ , the financial frictions on the household side make the steady-state interest rate  $R_b$  lower than  $1/\beta$ . The presence of borrowing constraints limits households' ability to diversify the uninsurable risk  $\theta$ , thus inducing households to over-save to self-insure against risks. The oversaving behavior consequently reduces the interest rate in equilibrium.

Now, combining equations (A.20), (A.23) and (A.30), and with some algebra, we have

$$(1 - \alpha) \left[ (1 - u) + \kappa u^f \right] Y / \bar{K} = (\Upsilon - R_b + 1) q + \Upsilon q b \quad (\text{A.32})$$

$$+ (\Upsilon - R_b + 1) S / K + \left\{ \Upsilon + 1 - \left[ \tau / (1 + \tau) R_b^f + 1 / (1 + \tau) R_b \right] \right\} \tilde{S} / K,$$

where  $\Upsilon = 1/(1/\Psi - 1)$ ,  $\bar{K} = u^f K^f + (1 - u)K$  is total world capital stock employed by the home country and  $\kappa \equiv K^f/K$  is the relative ratio of fixed capital stocks in the two countries.

Equation (A.32) describes the aggregate supply of capital (capital-to-output ratio) for the home country as a positive function of the interest rate. Given  $u^f$  and  $\kappa$ , the LHS of the equation is decreasing in  $\bar{K}/Y$  since both  $1 - u$  and  $Y/\bar{K}$  are decreasing in  $\bar{K}/Y$ ; whereas given  $R_b^f$  (and ignoring the terms  $S$  and  $\tilde{S}$  for simplicity), the RHS of (A.32) is decreasing in  $R_b$  since both  $\Psi$  and  $q$  are decreasing functions of  $R_b$ .<sup>42</sup>

## A.5 Proof of Proposition 2

The proof proceeds in two steps. First, we show that there exist parameter values of financial development such that the home country in autarky has higher MPK and a lower interest rate. Then we show that under these parameter values, the home country, in financial liberalisation, holds a positive position in financial capital and a negative position in fixed capital.

**Lemma 1** *Suppose the home country has tighter borrowing constraints on the firm side, that is,  $\xi^h < \xi^f$ , then for any  $b^f$ , there exist  $\underline{b}$  and  $\bar{b}$  such that if  $b^h \in (\underline{b}, \bar{b})$ ,<sup>43</sup> in the financial autarky regime, the home country has higher MPK and a lower real interest rate.*

**Proof of Lemma 1.** In the financial autarky regime, the equilibrium return of capital  $r^\ell$  (MPK) (or the inverse  $\frac{K}{Y}$  ratio) and the real interest rate  $R_b^\ell$  are determined by (29) and (30). In the autarky equilibrium  $r^\ell$  and  $R_b^\ell$  are the functions of financial developments  $\{\xi^\ell, b^\ell\}$ , which we denote as  $r_{Aut}(\xi^\ell, b^\ell)$  and  $R_{Aut}^*(\xi^\ell, b^\ell)$  respectively. As Proposition 7 shows, we have  $\partial r_{Aut}(\xi^\ell, b^\ell)/\partial b^\ell > 0$ ,  $\partial r_{Aut}(\xi^\ell, b^\ell)/\partial \xi^\ell < 0$  and  $\partial R_{Aut}^*(\xi^\ell, b^\ell)/\partial b^\ell > 0$ . Therefore, there exists  $\bar{b}$  satisfying

$$R_{Aut}^*(\xi^f, b^f) = R_{Aut}^*(\xi^h, \bar{b}), \quad (\text{A.33})$$

such that for any  $b^h < \bar{b}$ , we must have  $R_{Aut}^*(\xi^f, b^f) > R_{Aut}^*(\xi^h, b^h)$ .<sup>44</sup> There also exists  $\underline{b}$  satisfying

$$r_{Aut}(\xi^f, b^f) = r_{Aut}(\xi^h, \underline{b}), \quad (\text{A.34})$$

<sup>42</sup>Here we implicitly assume that the term  $1/(1 - \Psi) - R_b$  is strictly positive. Indeed, this assumption holds under fairly weak conditions. To see this, since  $1/(1 - \Psi) - R_b > 0$  implies  $1 - \Psi < 1/R_b$ , from equation (A.31), we only need to show  $1 - \Psi < \beta G$ . According to the definitions of  $\Psi$  and  $G$ , the last inequality is equivalent to  $(1 - \beta)F(\bar{\theta}) < \int_{\theta < \bar{\theta}} \theta/\bar{\theta} dF + \beta \int_{\theta > \bar{\theta}} \theta/\bar{\theta} dF$ . Therefore, we only need to show  $F(\bar{\theta}) < \beta/(1 - \beta) \left( \int_{\theta < \bar{\theta}} \theta/\bar{\theta} dF + \int_{\theta > \bar{\theta}} \theta/\bar{\theta} dF \right)$ . This inequality always holds if we have  $E(\theta)/\bar{\theta} \geq (1 - \beta)/\beta$ , which is easily satisfied under the conditions that  $\beta \rightarrow 1$  and  $b$  is not too large (to ensure  $\theta \ll \theta_{\max}$ ).

<sup>43</sup>Of course,  $\bar{b}$  and  $\underline{b}$  are the functions of  $\xi^h, \xi^b$  and  $b^f$ . Without risk of confusion, here we do not express them explicitly as  $\bar{b}(\xi^h, \xi^f, b^f)$  and  $\underline{b}(\xi^h, \xi^f, b^f)$ .

<sup>44</sup>If we assume the investment efficiency shock follows Pareto distribution, it can be shown that the  $R_{Aut}^*$  does not depend on  $\xi$ , therefore  $\bar{b}$  is simply  $b^f$ . That is, the higher level of financial development of foreign country on the household side induces a higher interest rate.

such that for any  $b^h > \underline{b}$ ,<sup>45</sup> we must have  $r_{Aut}(\xi^h, b^h) > r_{Aut}(\xi^f, b^f)$ . Note that  $\bar{b} > \underline{b}$  because of  $\partial \mathbb{R}(R_b^\ell, \xi^\ell) / \partial R_b > 0$  and  $\partial \mathbb{R}(R_b^\ell, \xi^\ell) / \partial \xi < 0$ .

**Lemma 2** *Suppose the home country has tighter borrowing constraints on the household side, i.e.  $b^h < b^f$ , and idiosyncratic investment efficiency  $\varepsilon$  follows the Pareto distribution, then for any  $\xi^f$ , there exists  $\bar{\xi}$  such that if  $\xi^h < \bar{\xi} < \xi^f$ , in the financial autarky regime, the home country has higher MPK and a lower real interest rate.*

**Proof of Lemma 2.** From (A.29), the Pareto distribution of  $\varepsilon$  implies the MPK (or  $Y/K$ ) in the autarky regime is a linear function of  $q$ . Furthermore, (30) implies  $R_{Aut}^*$  depends only on  $b$  and not on  $\xi$ . Thus, according to Proposition 6,  $b^h < b^f$  implies  $R_{Aut}^*(b^h) < R_{Aut}^*(b^f)$ . On the other hand, similar to the proof of Lemma 1, there exists  $\bar{\xi}(\xi^f, b^f, b^h)$  satisfying

$$r_{Aut}(\xi^f, b^f) = r_{Aut}(\bar{\xi}, b^h) > r_{Aut}(\xi^f, b^h) \quad (\text{A.35})$$

such that for any  $\xi^h < \bar{\xi} < \xi^f$ , we must have  $r_{Aut}(\xi^h, b^h) > r_{Aut}(\xi^f, b^f)$ .

We now turn to proving Proposition 2 with one of the following conditions: (i)  $\xi^h < \xi^f$  and  $b^h \in (\underline{b}, \bar{b})$ , as stated in Lemma 1; or (ii)  $b^h < b^f$ ,  $\xi^h < \bar{\xi} < \xi^f$  and  $\varepsilon$  follows the Pareto distribution, as stated in Lemma 2. The pattern of two-way capital flows requires us to show that in the liberalisation regime interest rates satisfy  $R_b^f > R_b^h$  and MPKs satisfy  $r^h > r^f$ . We proceed with the proof by ruling out all the complementarity relationships.

First, we show  $R_b^f = R_b^h$  is impossible. In this case, there is no financial capital flow across countries. Since  $\mathbb{R}(R_b, \xi)$  is decreasing in  $\xi$  and  $\xi^h < \xi^f$ , we must have  $R_k^h > R_k^f$  and  $r^h > r^f$ .<sup>46</sup> The higher MPK in home country attracts FDI from foreign country, i.e.  $u^f > 0$ . FDI inflow will shift the capital supply downward.<sup>47</sup> Consequently, FDI inflow reduces the interest rate in the home country and raises the interest rate in the foreign country. This means that unlike the autarky equilibrium, we have  $R_b^f > R_{Aut}^*(\xi^f, b^f) > R_{Aut}^*(\xi^h, b^h) > R_b^h$ , which contradicts  $R_b^f = R_b^h$ .

Second, we show  $r^h = r^f$  is impossible. In this case, there is no FDI flow across countries, and  $R_k^h = r^h = R_k^f = r^f$ . Under the parameter values satisfying Lemma 1 or 2, we must have  $R_b^h < R_b^f$  since  $\mathbb{R}(R_b, \xi)$  is decreasing in  $\xi$  and increasing in  $R_b$ .<sup>48</sup> The higher interest rate in the foreign country attracts bond inflow, which shifts the capital supply curve in the foreign country

<sup>45</sup>Note that since  $\xi^h < \xi^f$ ,  $\partial r_{Aut}(\xi^\ell, b^\ell) / \partial \xi^\ell < 0$  implies  $\underline{b} < b^f$ .

<sup>46</sup>Thus, we have already ruled out two combinations:  $R_b^f = R_b^h$  and  $r^h < r^f$  or  $R_b^f = R_b^h$  and  $r^h = r^f$ .

<sup>47</sup>More specifically, in this case, the capital supply curve in the home country (Eqn. A.32) takes the form  $(1 - \alpha)(1 + \kappa u^f) Y^h / \bar{K}^h = [1 / (1 - \Psi^h) - R_b^h] q^h + 1 / (1 / \Psi^h - 1) q^h b^h$ . Since  $\kappa u^f > 0$ , compared with the autarky regime, the supply curve shifts downward.

<sup>48</sup>Thus we rule out the combination  $r^h = r^f$  with  $R_b^h \geq R_b^f$ .

downward. In contrast, the bond outflow moves the capital supply curve in the home country upward.<sup>49</sup> As a result, compared to the autarky regime, MPK in the home country increases and MPK in the foreign country falls. Therefore we have  $r^h > r_{Aut}(\xi^h, b^h) > r_{Aut}(\xi^f, b^f) > r^f$ , which is a contradiction with  $r^h = r^f$ .

Third, we show  $R_b^f < R_b^h$  and  $r^h > r^f$  are impossible. In this case, the home country experiences both FDI and bonds inflows, both of which shift the capital supply curve downward and thus reduce the interest rate:  $R_b^h < R_{Aut}^*(\xi^h, b^h)$ . In contrast, the FDI and bonds outflows in the foreign country shift both capital demand and supply curves upwardly.<sup>50</sup> As a result, the interest rate in foreign country increases, i.e.,  $R_b^f > R_{Aut}^*(\xi^f, b^f)$ . Since with the parameter values satisfying Lemma 1 or 2, we have  $R_{Aut}^*(\xi^h, b^h) < R_{Aut}^*(\xi^f, b^f)$ , and thus we must have  $R_b^h < R_b^f$ , which contradicts  $R_b^f < R_b^h$ . With the same logic, we can show  $R_b^f > R_b^h$  and  $r^h < r^f$  are impossible as well.

Hence, the home country in the fully liberalisation regime has higher MPK and a lower interest rate. Consequently, the home country will hold a positive position in fixed capital and a negative position in financial capital. ■

<sup>49</sup>More specifically, in this case, the capital supply curve in the home country (Eqn. A.32) takes the form  $(1 - \alpha)Y^h/\bar{K}^h = [1/(1 - \Psi^h) - R_b^h]q^h + 1/(1/\Psi^h - 1)q^hb^h + [1/(1 - \Psi^h) - (\tau R_b^f + R_b^h)/(1 + \tau)]\tilde{S}^h/K^h$ . Since the last term on the RHS is greater than zero, compared with the autarky regime, the supply curve shifts upward.

<sup>50</sup>In particular, the FDI outflow simultaneously shifts capital demand and supply upward. The bonds outflow only shifts the capital supply upward.